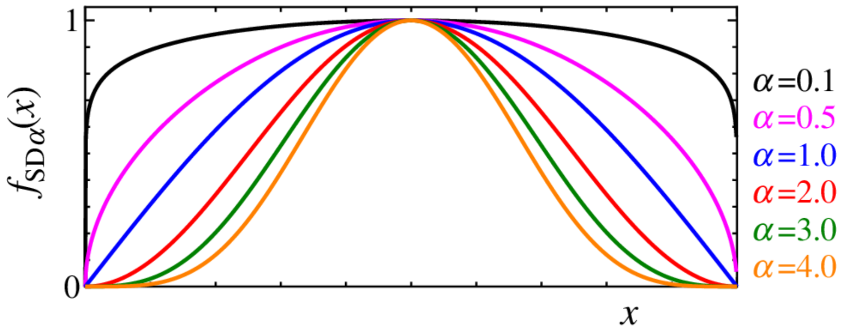
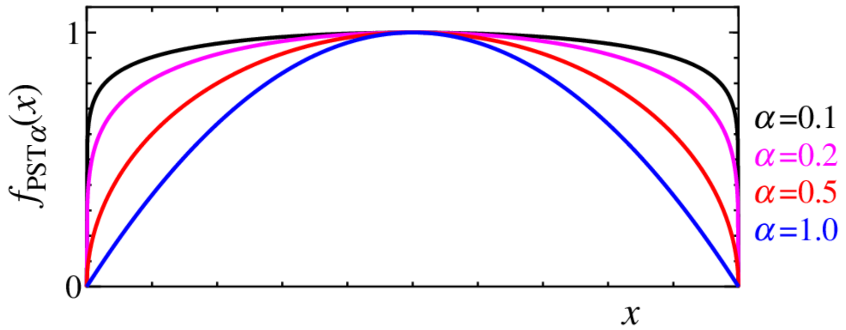
様々なエネルギースケール変換による一次元量子系の特性変化

引原 俊哉（群馬大）

Acknowledgement: 桂法称（東大）
上田宏（理研）
鈴木 隆史（兵県大）





11 Jul. 2019 @ SSD workshop, Riken

Outline

1. Introduction
   - energy-scale deformation
   - SSD

2. Sine-α deformation
   - Long-distance entanglement
   - Ground state properties of SDα chain
     - local energy density, entanglement entropy

3. PST-α deformation
   - Deformation for Perfect-State Transfer
   - Ground state properties of PSTα chain
     - local energy density, entanglement entropy

Energy-scale deformation

$$\mathcal{H} = \sum_x f(x)h(x)$$

$f(x)$: scale function $h(x)$: hamiltonian density

We will consider the spin-1/2 XXZ chain

$$h(x_j) = S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z$$

$\Delta = 0$: XX chain, free fermion

$\Delta \neq 0$: interacting

Uniform XXZ chain

$$\mathcal{H} = \sum_j h(x_j) = \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\right)$$

Ground state (low-energy) properties
of the spin-1/2 XXZ chain in critical region
are well known
for thermodynamic limit / periodic b.c. / open b.c.

e.g.) for the uniform chain under open b.c.

$$\langle S_j^x S_k^x \rangle = A^x (-1)^{j-k} \frac{[g(j)g(k)]^{\eta/2}}{[g(\frac{j-k}{2})g(\frac{j+k}{2})]^\eta} + \cdots$$

$$d^a(x_j) = \langle S_j^a S_{j+1}^a \rangle = C_0^a + \frac{(-1)^j C_1^a}{[g(x_j)]^{1/2\eta}} + \cdots$$

$$g(x_j) = \frac{L+1}{\pi} \sin\left(\frac{\pi x_j}{L+1}\right)$$

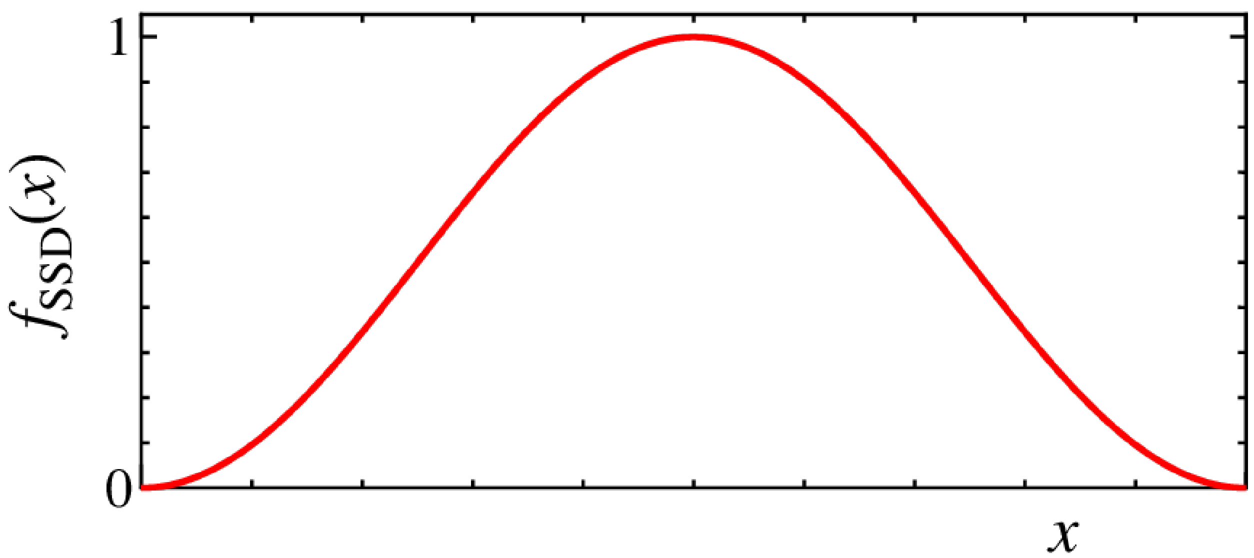
◆ SSD

Sine-Square Deformation (SSD)

Gendiar et al. (2009)

$$\mathcal{H}_{\mathrm{SSD}} = \sum_x f_{\mathrm{SSD}}(x) h(x)$$

$$f_{\mathrm{SSD}}(x) = \sin^2\left(\frac{\pi(x-\frac{1}{2})}{L}\right)$$



SSD for 1d critical system

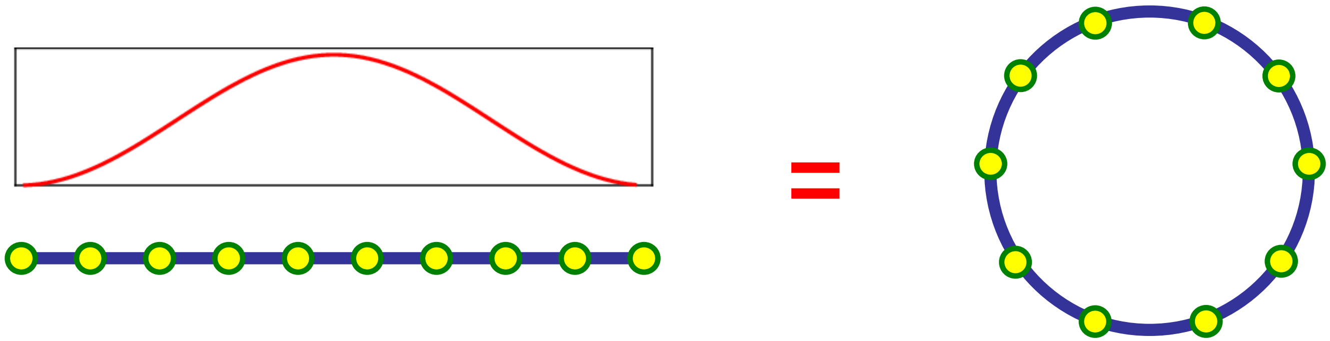
1d critical (gapless) systems under SSD

ground state is equivalent with that of periodic system

Hikihara-Nishino (2012)

Katsura (2011), Maruyama et al. (2011)

Hotta-Shibata(2011)



conformal mapping to infinite uniform chain

Wen-Ryu-Ludwig (2016)

grand-canonical analysis for magnetization curve

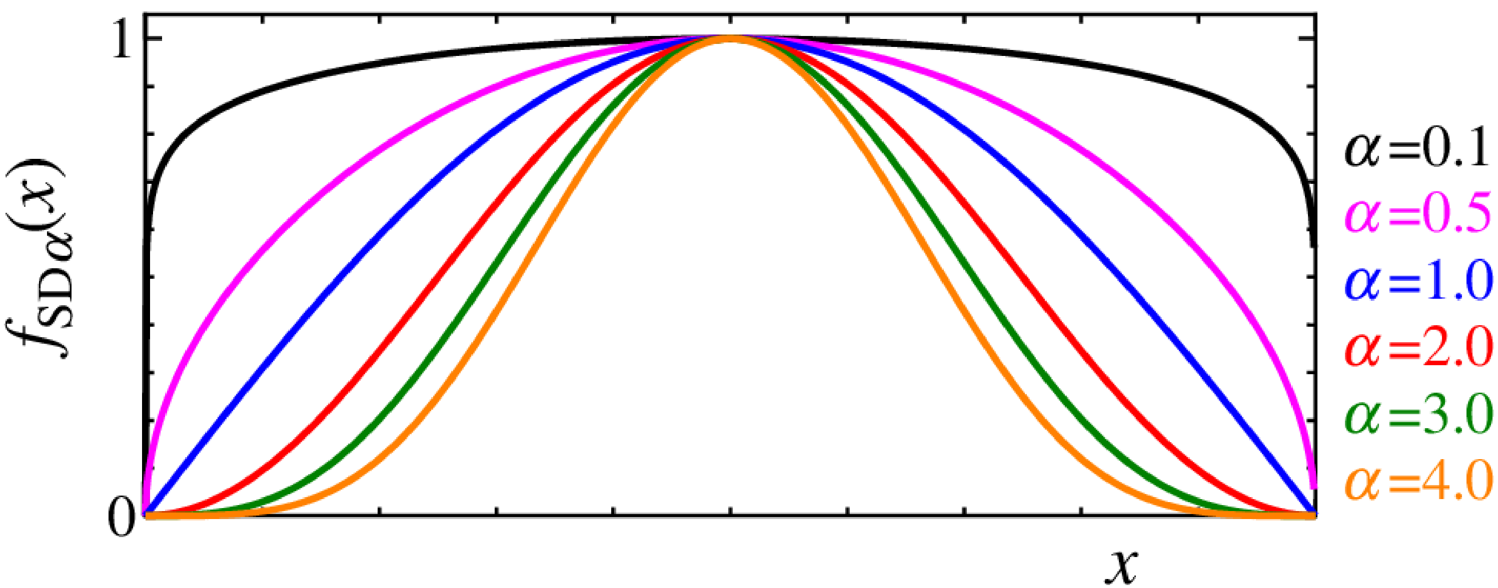
Hotta-Shibata(2012), Nishimoto et al.(2013)

Sine-α deformation (SDα)

Gendiar et al. (2009)

$$\mathcal{H}_{\mathrm{SD}\alpha} = \sum_x f_{\mathrm{SD}\alpha}(x) h(x)$$

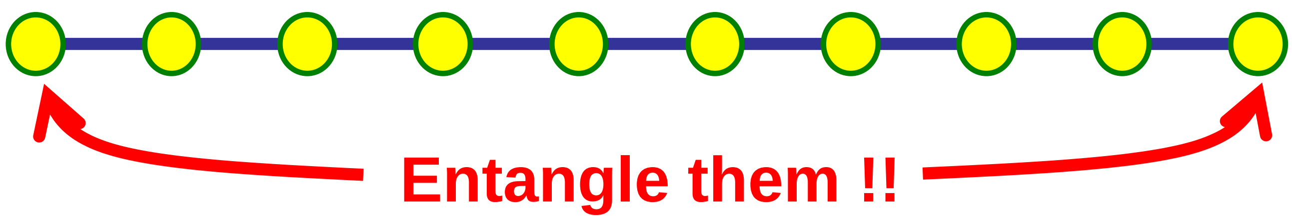
$$f_{\mathrm{SD}\alpha}(x) = \sin^\alpha \left(\frac{\pi\left(x - \frac{1}{2}\right)}{L} \right)$$



◆ Long Distance Entanglement

For realizing Quantum-Information process,
genelation of large entanglement
between qubits located at a large distance
and connected by steady channel
is desirable

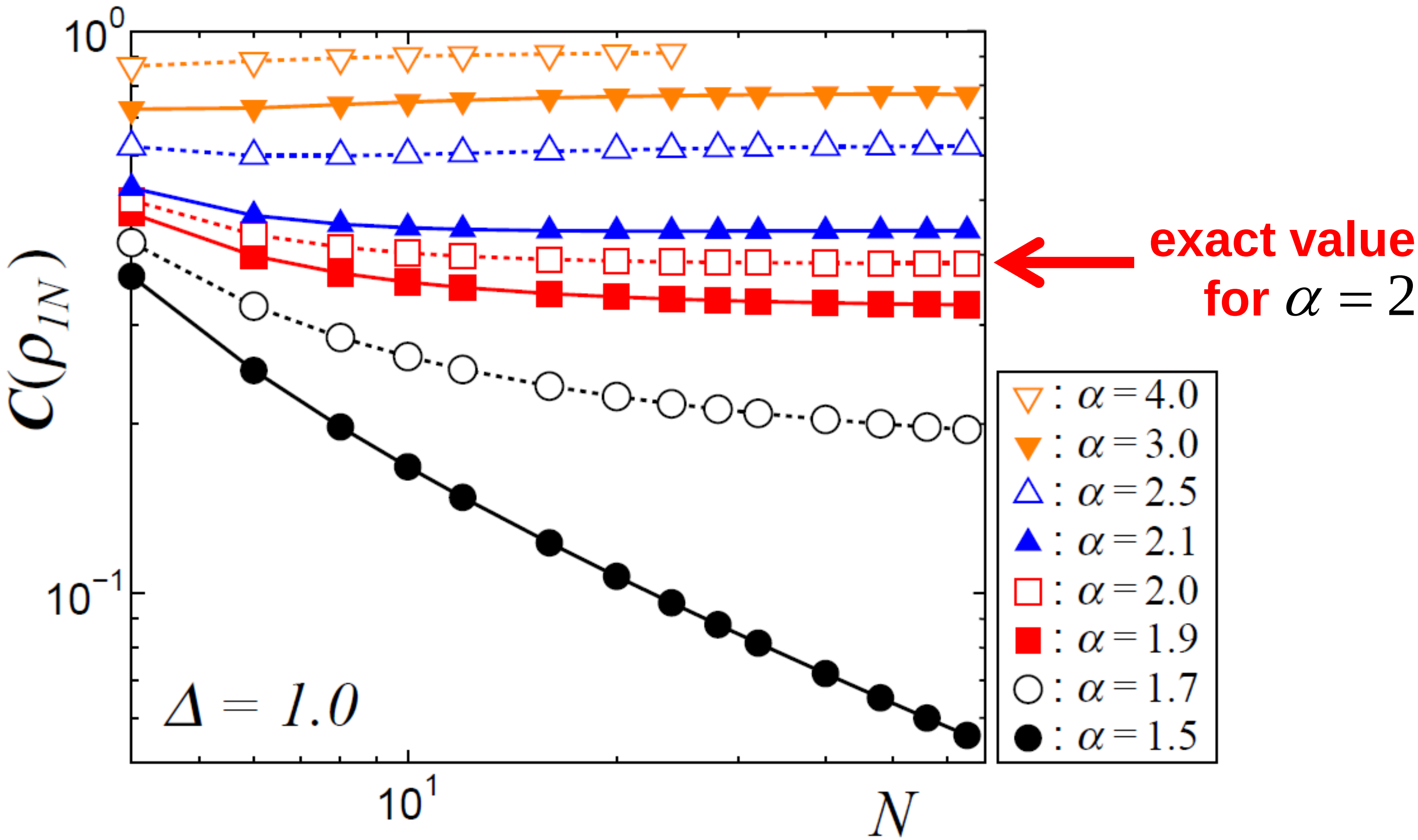
End-to-end entanglement in 1D quantum systems



SSD system realize true Long-Distance Entanglement

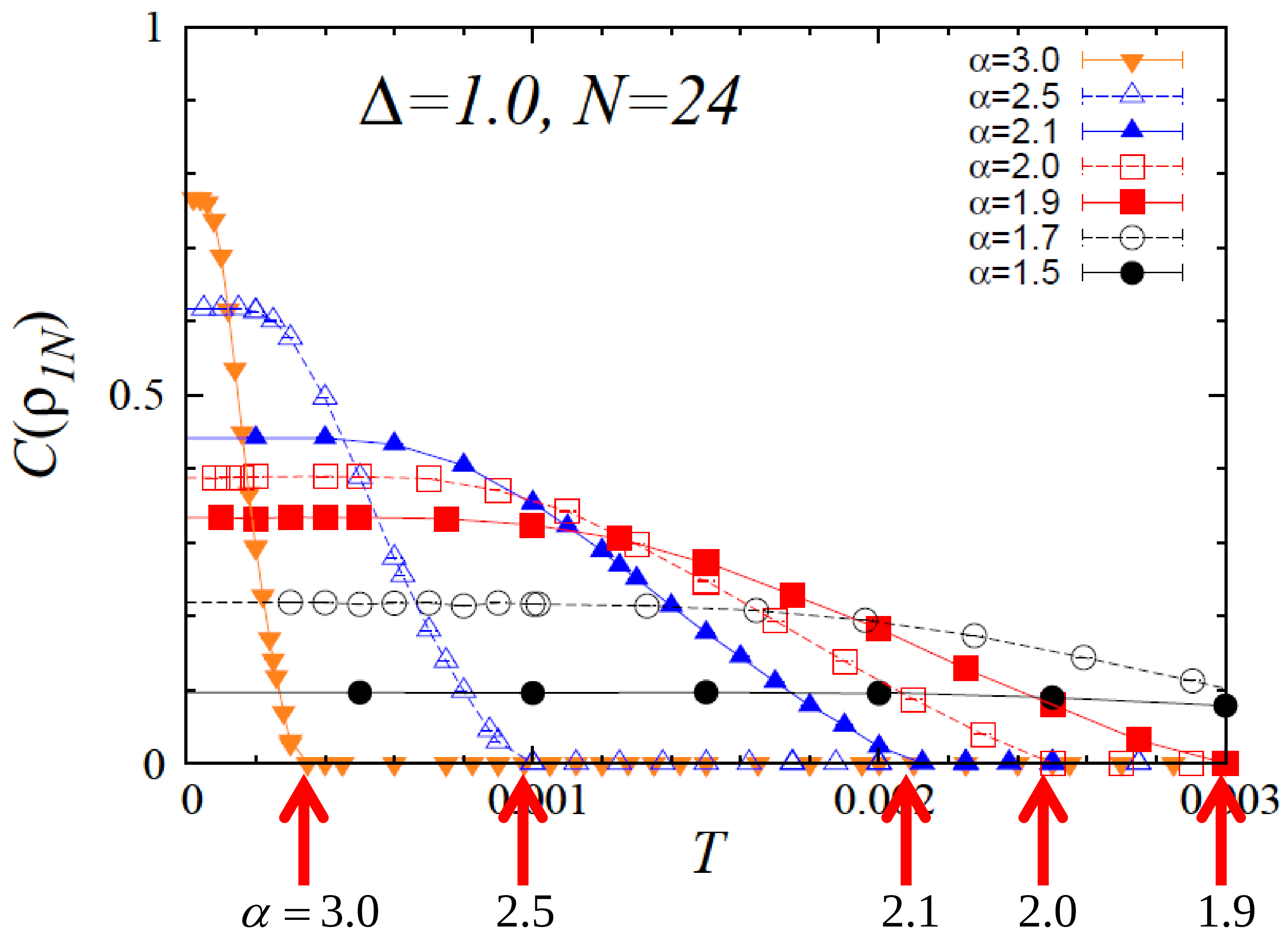
How end-to-end entanglement in SDα chain
depends on system size, temperature, …

End-to-end entanglement at T=0



- End-to-end concurrence for $\alpha \geq 2$ converges a finite value at $N \to \infty$ **: true LDE**
- Concurrence is larger as α is larger (coupling constants around edges are smaller)

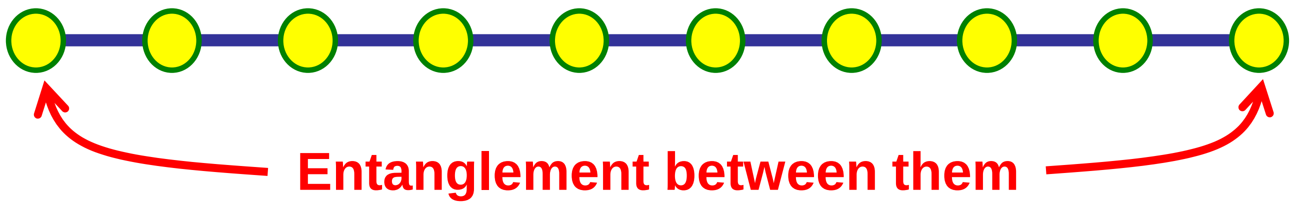
◆ End-to-end entanglement at finite Temperatures



- Critical temperature T^* is smaller as α is larger (coupling constants around edges are smaller)

◆ Long Distance Entanglement

End-to-end entanglement in 1D quantum systems



$$\mathcal{H}_{\mathrm{SD}\alpha} = \sum_x f_{\mathrm{SD}\alpha}(x) h(x) \qquad f_{\mathrm{SD}\alpha}(x) = \sin^{\alpha}\left(\frac{\pi(x-\frac{1}{2})}{L}\right)$$

true LDE realizes for $\boldsymbol{\alpha \geq 2}$

As α is larger,

end-to-end entanglement at T=0 larger

critical temperature T* smaller (LDE becomes fragile)

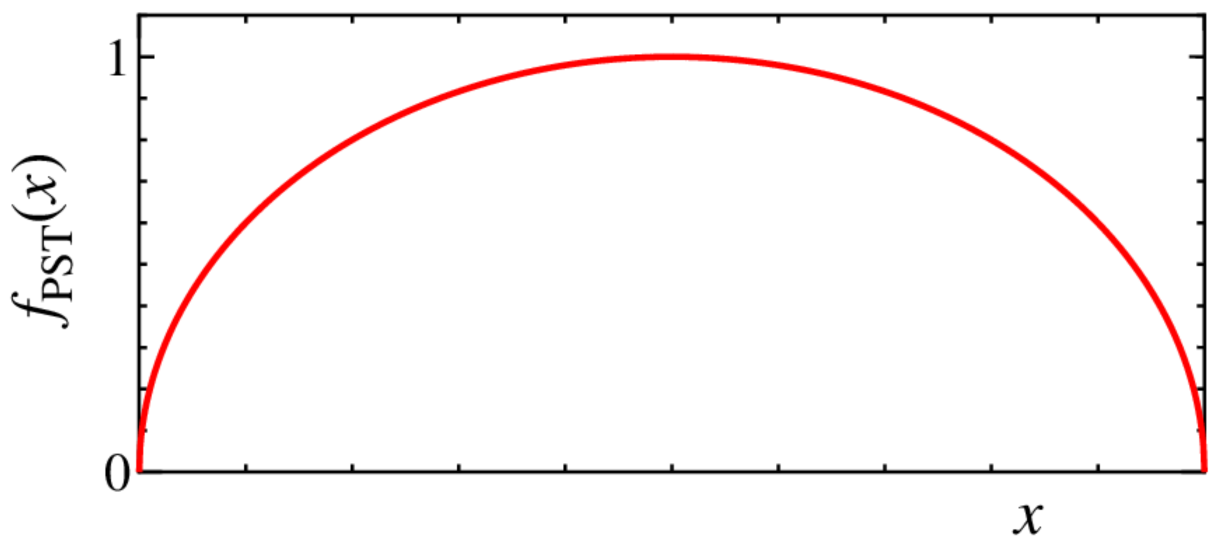
◆ Perfect-State Transfer

Energy-scale deformation for Perfect-State Transfer

Christandl et al. (2004)

$$\mathcal{H}_{\mathrm{PST}} = \sum_x f_{\mathrm{PST}}(x) h(x)$$

$$f_{\mathrm{PST}}(x) = \sqrt{\left(x - \frac{1}{2}\right)\left(L - x + \frac{1}{2}\right)}$$



◆ Perfect-State Transfer

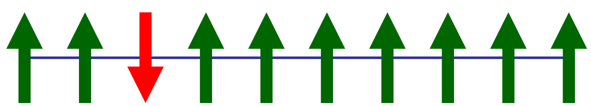
Perfect-State Transfer in XX-spin chain

Christandl et al. (2004)

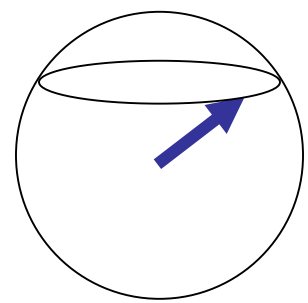
$$\mathcal{H}_{\rm PST} = \sum_{j=1}^{L-1} \sqrt{j(L-j)}\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right)$$

L-site system in one-magnon subspace

Single $S = (L-1)/2$ spin in transverse field



$$|j\rangle = S_j^- |\mathrm{all}\uparrow\rangle$$



$$|S^z = j\rangle$$

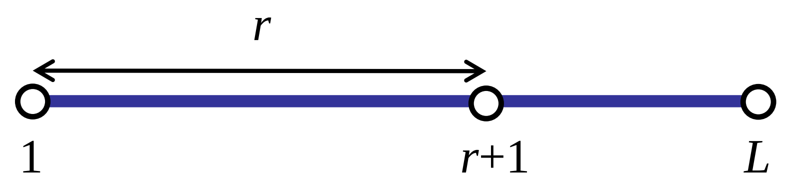
hopping amplitude

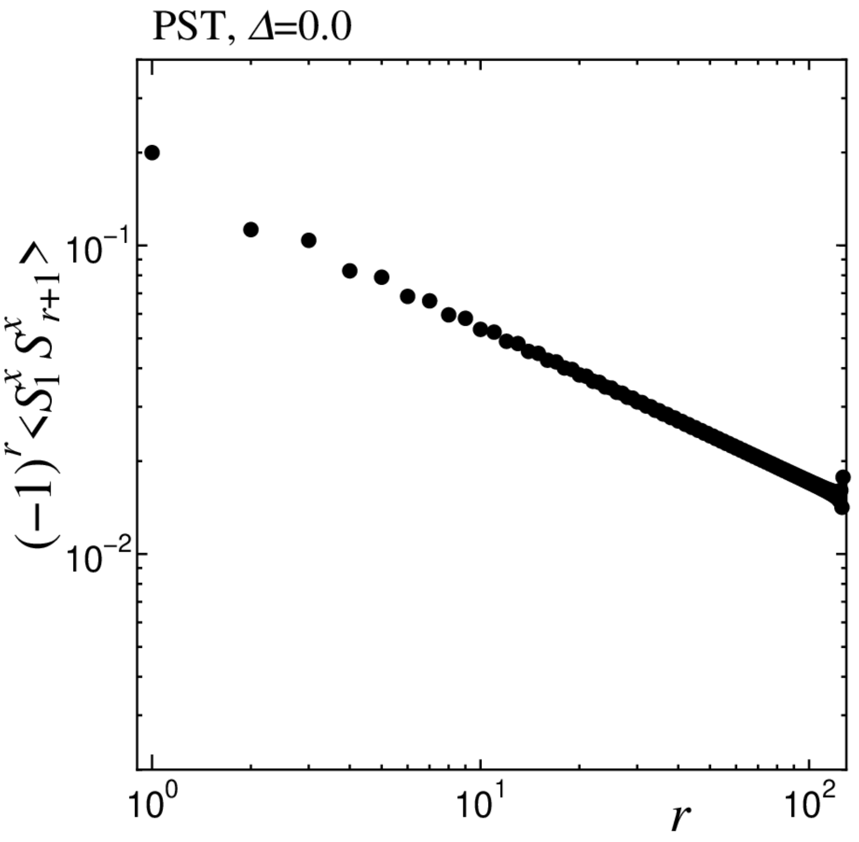
Clebsch-Gordan coeff.

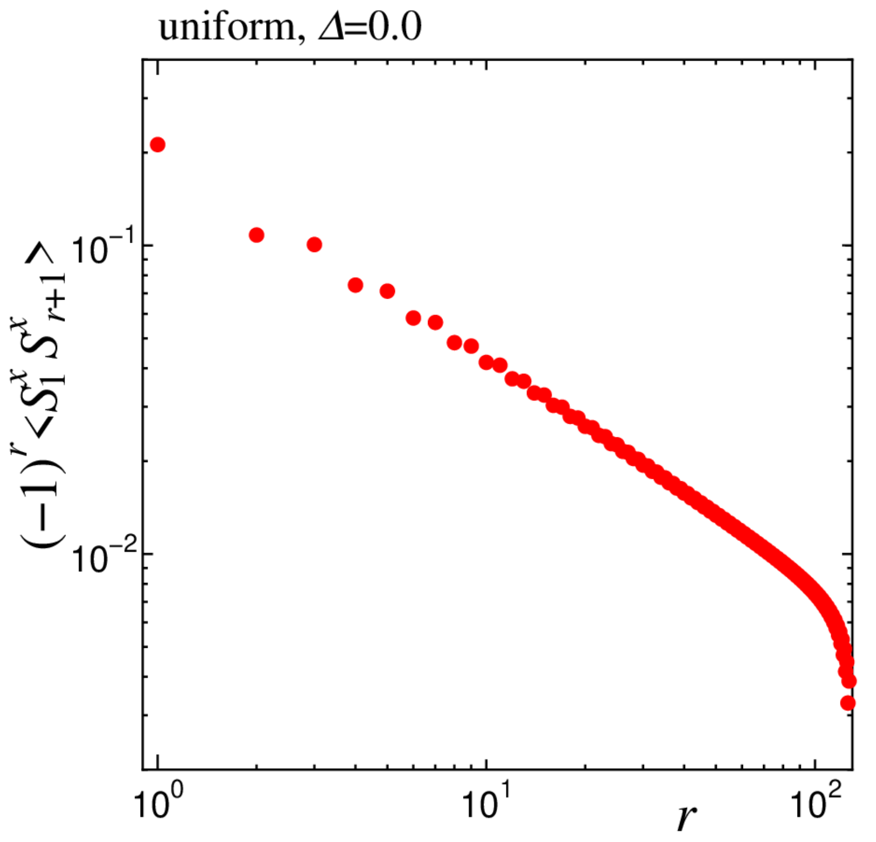
Quantum-state transfer without loss of fidelity

Constant level spacing of eigenenergies

◆ Transverse spin correlation in PST system







◆ CFT from open chain to PST system

Inverse sine mapping

Wen-Ryu-Ludwig (2016)

$$z = R \sin \omega$$

ω plane		z plane
$-\frac{\pi}{2} < u < \frac{\pi}{2}, \quad v = 0$	↔	$-R < x < R, \quad y = 0$
uniform chain		PST chain

leading term in staggered part of $\langle S_j^x S_k^x \rangle$

$$\frac{A^x (-1)^{j-k} \left[g(u_1) g(u_2) \right]^{\eta/2}}{\left[g(\frac{u_1 - u_2}{2}) g(\frac{u_1 + u_2}{2}) \right]^{\eta}} \quad \longleftrightarrow \quad \frac{A^x (-1)^{j-k}}{|x_1 - x_2|^{\eta}}$$