SSD@理研:2019/7/11

XXZ鎖における格子Unruh効果と世界線エンタングルメント

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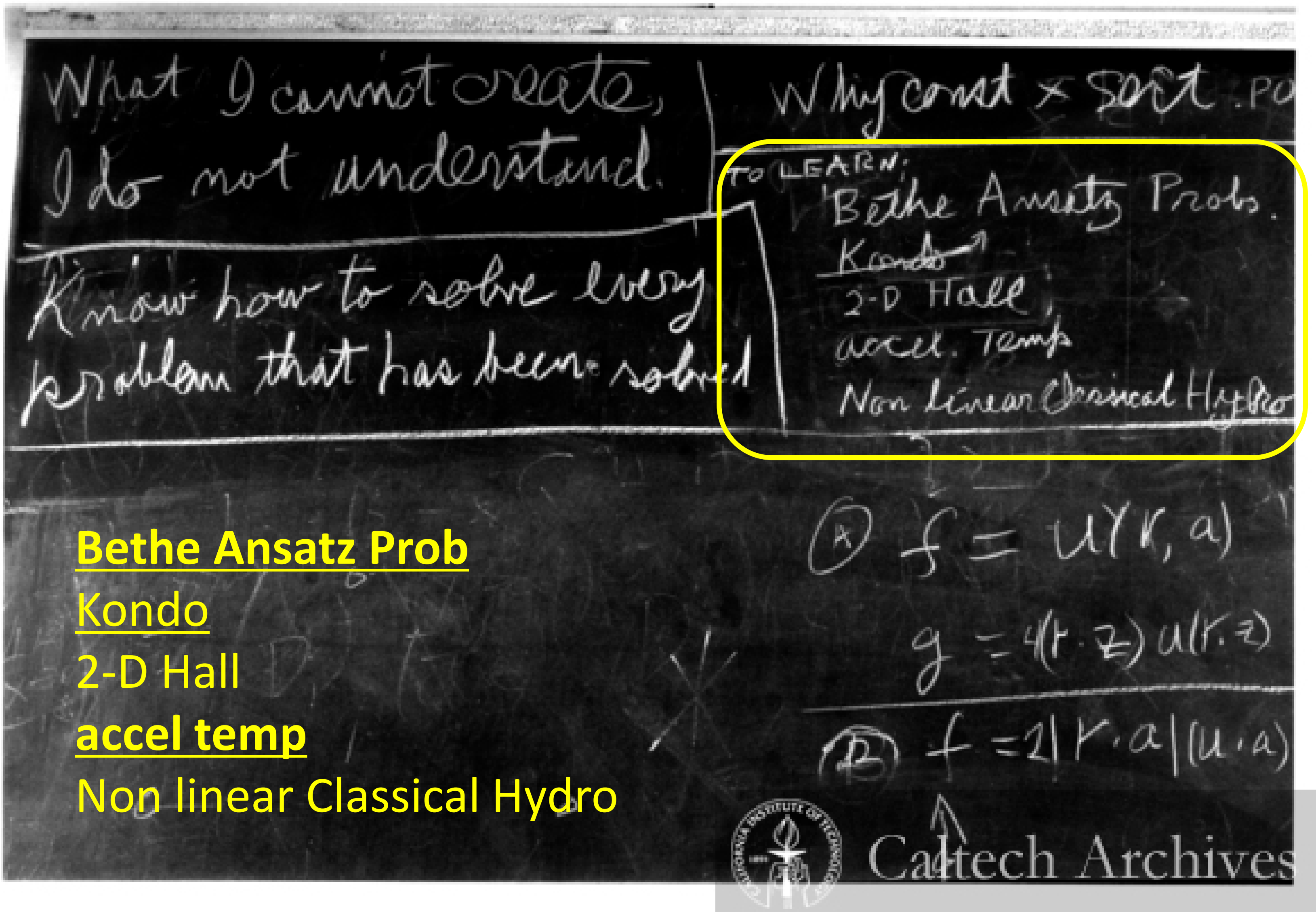
Feynman's blackboard at 1988



Bethe Ansatz Prob

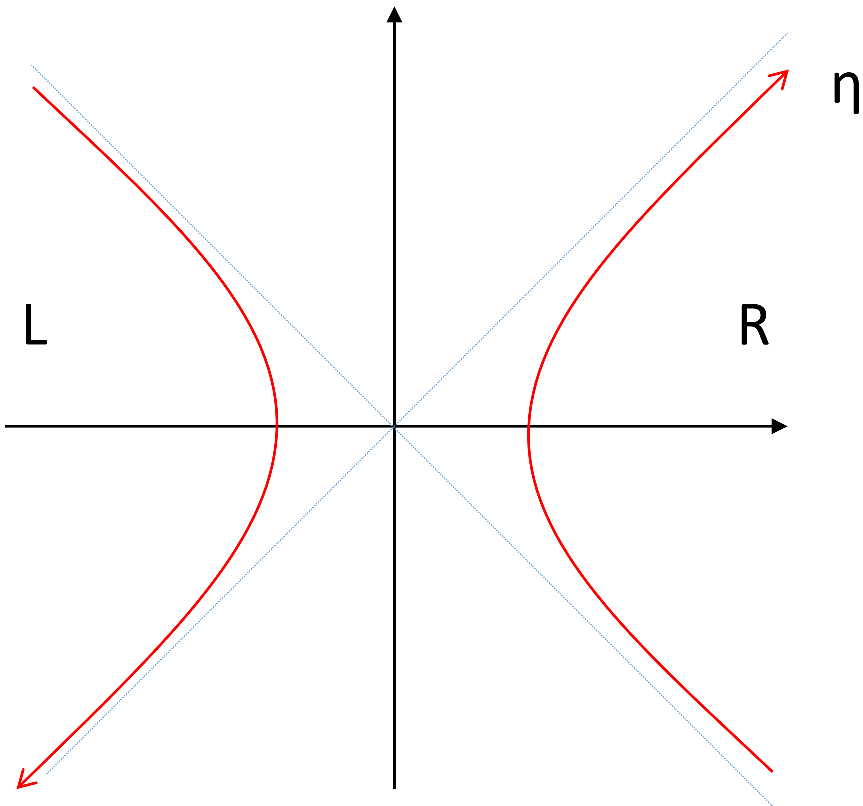
Kondo

2-D Hall

accel temp

Non linear Classical Hydro

Unruh effect



A constantly accelerating observer

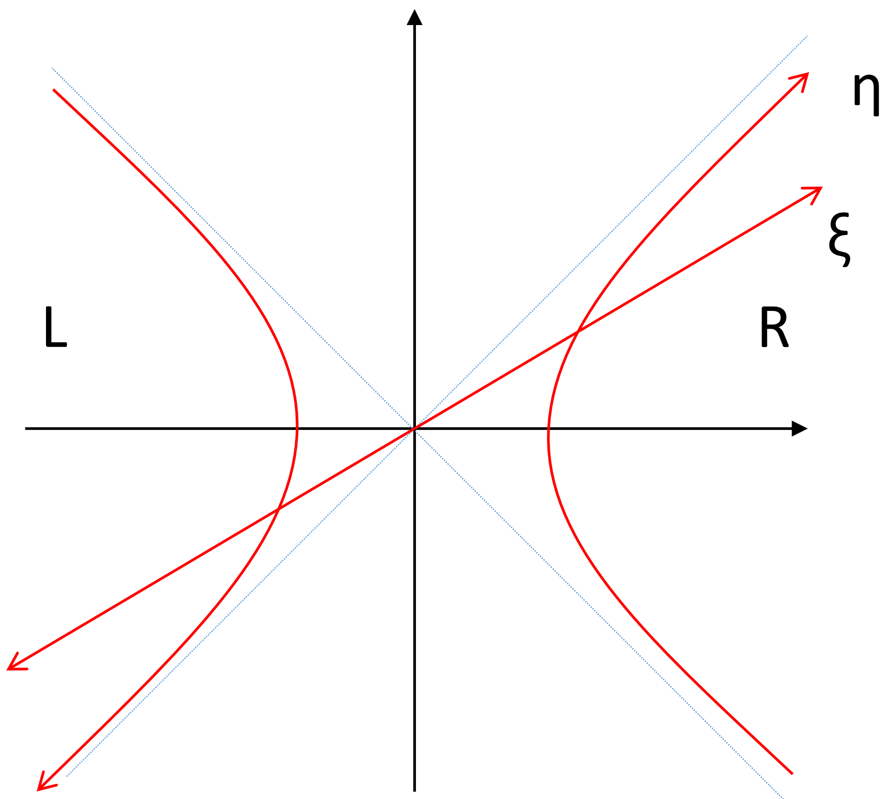
$$x = \frac{e^{a\xi}}{a}\cosh(a\eta)$$
$$t = \frac{e^{a\xi}}{a}\sinh(a\eta)$$

sees the vacuum as a thermalized state with an effective temp. (Unruh temp.)

$$\beta^* = \frac{2\pi}{a}$$

The Left and right parts are space like regimes, which are classically separable!

Rindler-Fulling quantization



constantly accelerating observer

R $$x = \frac{e^{a\xi}}{a}\cosh(a\eta)$$
$$t = \frac{e^{a\xi}}{a}\sinh(a\eta)$$

$a_k\,,a_k^\dagger$ with $a_k|0\rangle_M$

Minkowski vacuum

$b_p^{\mathrm{R}}\,,{b^{\mathrm{R}}}^\dagger_p\,,b_p^{\mathrm{L}}\,,{b^{\mathrm{L}}}^\dagger_p$ with

$$b_p^{\mathrm{R}}|0\rangle_R = b_p^{\mathrm{L}}|0\rangle_L = 0$$

<u>Bogoliubov transformation</u>

$$|0\rangle_M = e^{-\prod_p e^{-\pi p/a}{b^L}^\dagger_p{b^R}^\dagger_p}|0\rangle_L|0\rangle_R$$

$$\rho_R = \mathrm{Tr}_L|0\rangle_M\langle 0|_M = \prod_p e^{-\beta^* H_p}$$

with $\beta^* = \dfrac{2\pi}{a}$ and $H_p = p{b^R}^\dagger_p b_p^R$

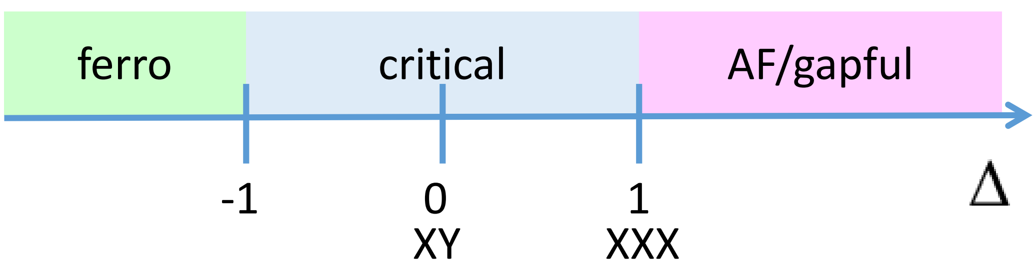
Ising-like XXZ chain

$$\lambda > 0 \quad (\Delta > 1)$$

$$\mathcal{H} = J_\lambda \sum_{n=-L+1}^{L} \left[S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z \right]$$

$$J_\lambda = \frac{2}{\sinh \lambda} \qquad \Delta = \cosh \lambda$$

The groundstate is gapful with a finite correlation length.

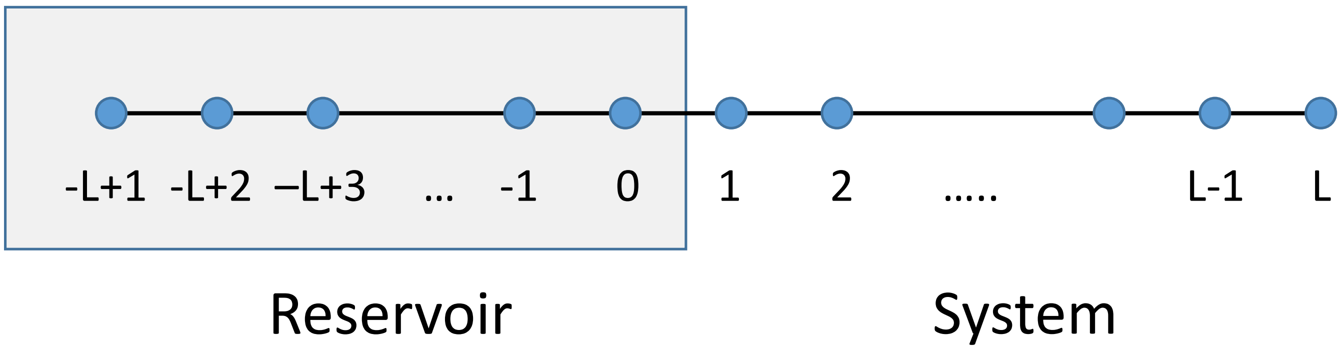


<u>Bethe ansatz solvable</u>

Bulk energy, excitation gap, magnetization, etc.

But, direct computation of the Bethe wavefunction is not so useful

geometry for EE



$$\rho = \sum_{n_-} \Psi^\dagger(n_-, n_+)\Psi(n_-, n_+) \quad \Rightarrow \quad S = -\mathrm{Tr}[\rho \log \rho]$$

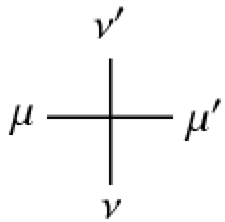
* This bipartition EE can be easily calculated by DMRG.

If we can write $\rho \sim \exp(-H_{\mathrm{EE}})$, H_{EE} is called entanglement Hamiltonian or modular Hamiltonian.

A modular Hamiltonian defines a time evolution in an angular direction different from the conventional time.

XXZ chain and 6-vertex model

$$W(\mu,\nu|\mu',\nu') =$$

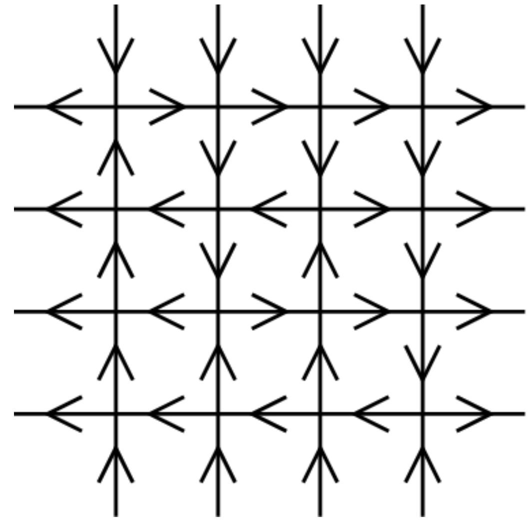


$$W(+,+|+,+) = W(-,-|-,-) = 1$$

$$W(+,-|-,+) = W(-,+|+,-) = \frac{\sinh(u)}{\sinh(\lambda - u)}$$

$$W(+,-|+,-) = W(-,+|-,+) = \frac{\sinh(\lambda)}{\sinh(\lambda - u)}$$

satisfies Yang-Baxter relation



$\lambda > 1$ Ising-like anisotropy = antiferroelectric regime

Commuting transfer matrices

$$[T(u), T(u')] = 0 \qquad T(u) = \sum_{\{\mu\}} \prod_n W_n(\mu_n, \nu_n|\mu_{n+1}, \nu_{n+1})$$

u : rapidity(=spectral parameter)

Hamiltonian of the XXZ chain $$\mathcal{H} = -\frac{d}{du}\log T(u)\Big|_{u=0}$$

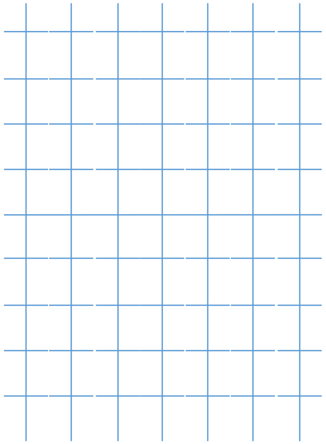
Simultaneous eigenstate $[T(u), \mathcal{H}] = 0$

integrability and CTM

Eigenvector: ~~Bethe type~~

Baxter's magic / CTM

$$|\Psi\rangle \sim \lim_{n\to\infty} T^n |b\rangle$$ =



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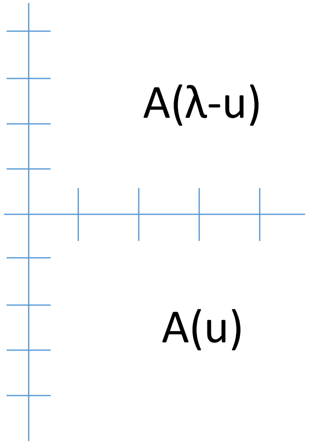


Baxter, J.Math.Phys. (1968), J.Stat.Phys. (1971)

The groundstate wavefunction of H can be written as a product of CTMs

$$\Psi \sim A(\lambda - u)A(u)$$ with $$A(u) \sim e^{-u\mathcal{K}}$$

$\mathcal{K}$ Hamiltonian of corner transfer matrix/corner Hamiltonian

$$\mathcal{K} \equiv J_\lambda \sum_{n=1}^{L} n \left\{ S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z \right\}$$

Lattice Lorentz boost operator
(Rapidity shift operator)

$$A(-\mu)T(\nu)A(\mu) = T(\mu + \nu)$$

$\Rightarrow$ The CTM formulation corresponds to the Rindler quantization of the relativistic quantum field theory

Lattice Poincare algebra

H.B.Thacker, Physica D **18**, 348 (1986).

$$[P, \mathcal{H}] = 0\,, \quad [\mathcal{K}, P] = iH\,, \quad [\mathcal{K}, H] = i\tilde{I}_2$$

$$I_0 = iP \quad I_1 = -\mathcal{H} \quad \tilde{I}_2 = iI_2 = \sum [h_{n,n+1}, h_{n+1,n+2}] \quad \log T(u) = \sum \frac{I_n}{n!} u^n,$$

<u>Reduced density matrix</u>

$\mathcal{K}$ plays a role of the entanglement Hamiltonian

$$\rho = \exp(-\beta_\lambda \mathcal{K})/Z$$

with

$$\beta_\lambda \equiv 2\lambda$$
$$Z \equiv \mathrm{Tr} \exp(-\beta_\lambda \mathcal{K})$$

entanglement/corner Hamiltonian

$$\mathcal{K} \equiv J_\lambda \sum_{n=1}^{L} n \left\{ S^x_n S^x_{n+1} + S^y_n S^y_{n+1} + \Delta S^z_n S^z_{n+1} \right\}$$

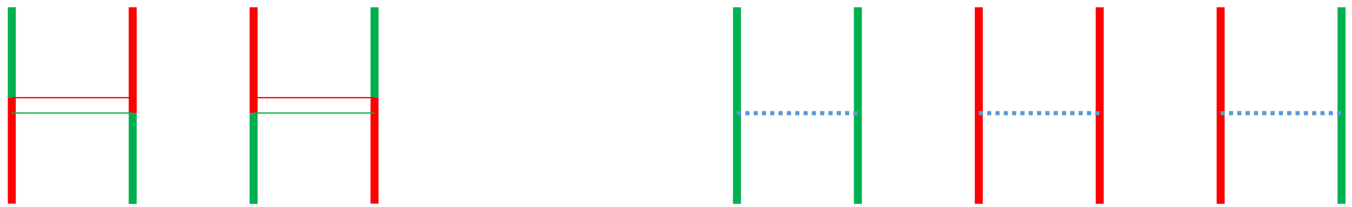
Free boundary condition at n=1, L

→ The boundary effect at n=1 should be perfectly suppressed

The energy scale is proportional to n

→ Effective temperature decreases as n increases.
(This can be a source of difficulty in a QMC simulation)

WL QMC



off-diagonal interaction
(XY-terms)

diagonal interaction
(zz terms)

The energy scale is proportional to n. ➡ We draw WLs as circles
"classical" entanglement surrounding the entangle point

Scale imaginary time: τ $\quad \theta = a\tau \quad$ with $\quad a = \dfrac{2\pi}{\beta_\lambda} = \dfrac{\pi}{\lambda}$

$0 \le \tau < \beta_\lambda$

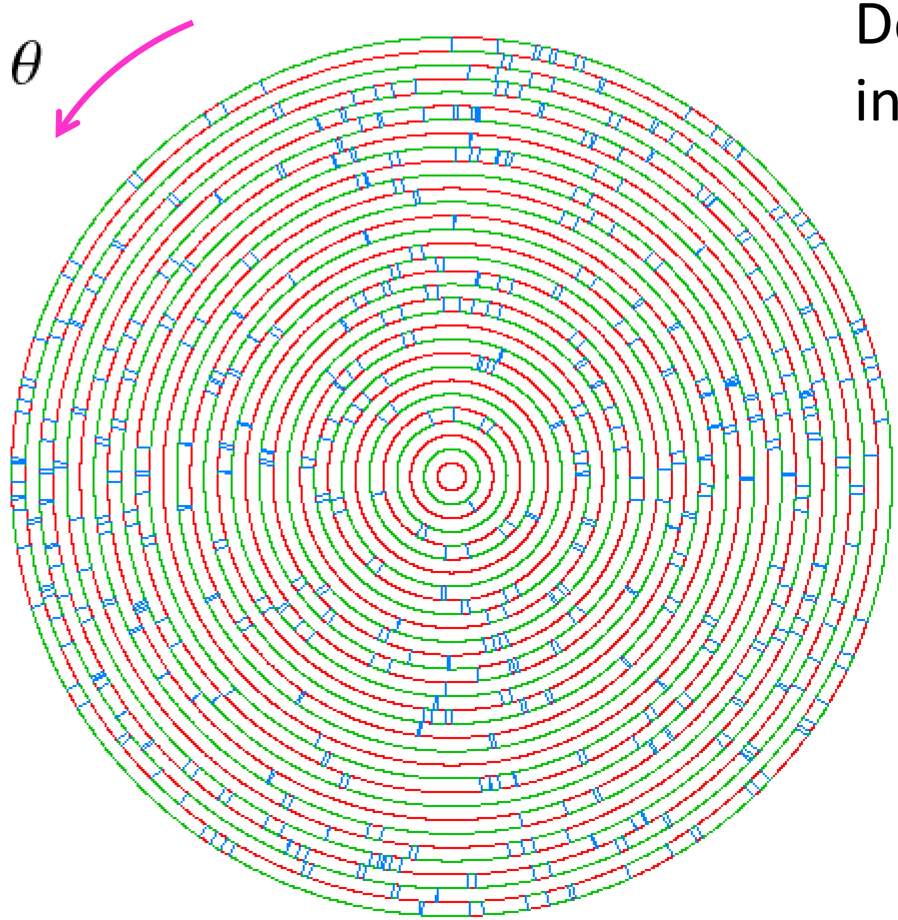
a : effective acceleration

a=0 : classical limit

snapshots

$\Delta = 2.0$ $(\lambda = 1.3169\cdots)$ $\beta_\lambda \equiv 2\lambda$

How can the "uniform" ground state be realized for the non uniform Hamiltonian?



Density of kinks looks uniform in this plot!

Local temp $\propto n\beta_\lambda$

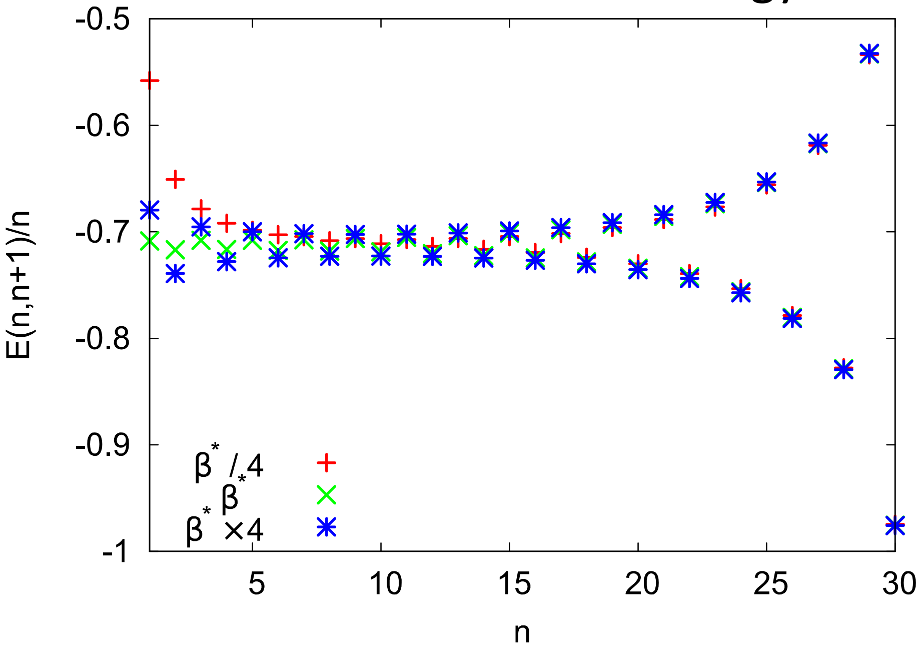
kink # $\propto n\beta_\lambda$

For $\beta < \beta_\lambda$ (high temp.), kinks around the center becomes space.

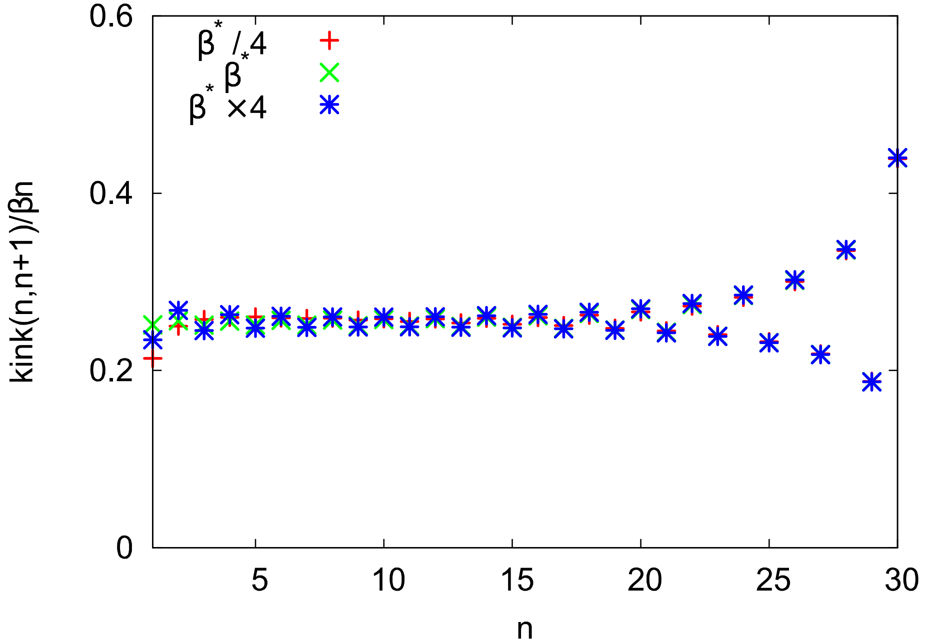
For $\beta > \beta_\lambda$(low temp.), kinks around the center are oscillating.

bond energy distribution $\Delta = 2.0$

$\langle S_n \cdot S_{n+1} \rangle$ normalized bond energy



normalized kink density

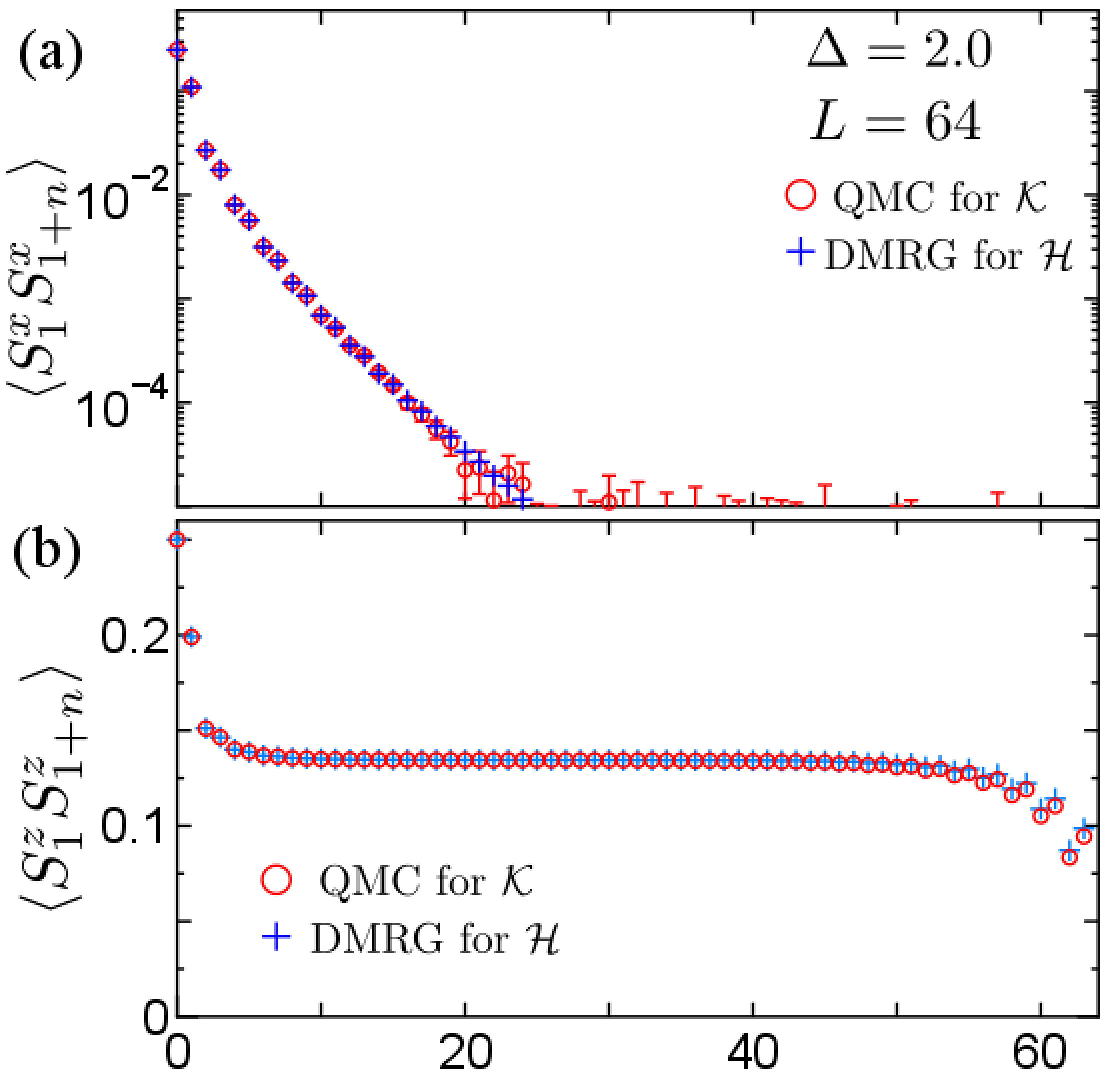


kink density is related to the off-diagonal parts of local energy

At $\beta = \beta^*$, the normalized bond energy and kink density become flat around n=1

⇒ reproduces uniform ground state wavefunction.

correlation functions $\Delta = 2.0$



Perfect correspondence to the DMRG results for the groundstate of H

Entanglement Entropy

The groundstate entanglement entropy for H can be calculated as the thermal entropy for the entanglement Hamiltonian.

$$S_{\mathrm{EE}} = -\mathrm{Tr}_S[\rho \log \rho] = \beta_\lambda \langle \mathcal{K} \rangle + \log Z$$

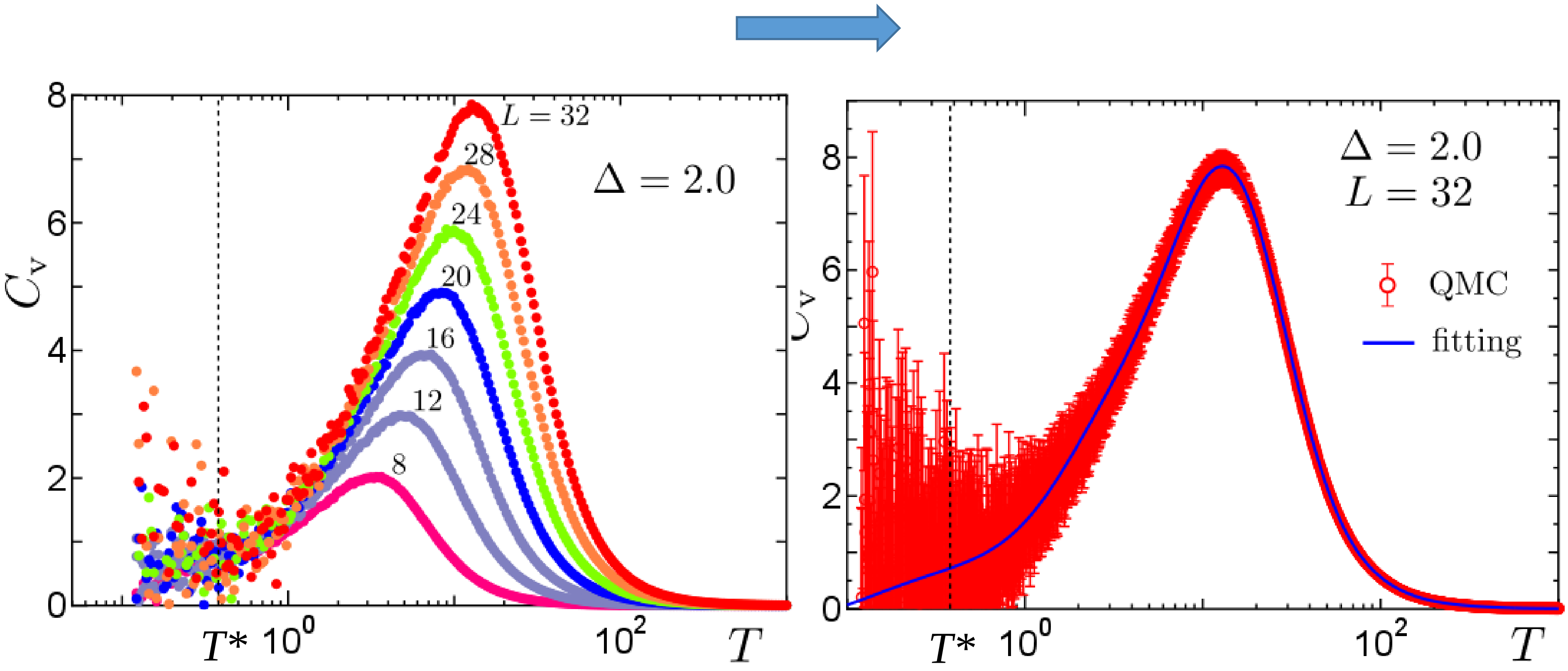
We calculate S_EE with integration of a specific heat estimated by a QMC simulation.

$$S_{\mathrm{EE}} = L \log 2 - \int_{T_\lambda}^{\infty} \frac{C_{\mathrm{v}}}{T} dT = L \log 2 - \int_{\log T_\lambda}^{\infty} C_{\mathrm{v}} dx$$

The estimation of the entropy is not easy but possible with QMC.

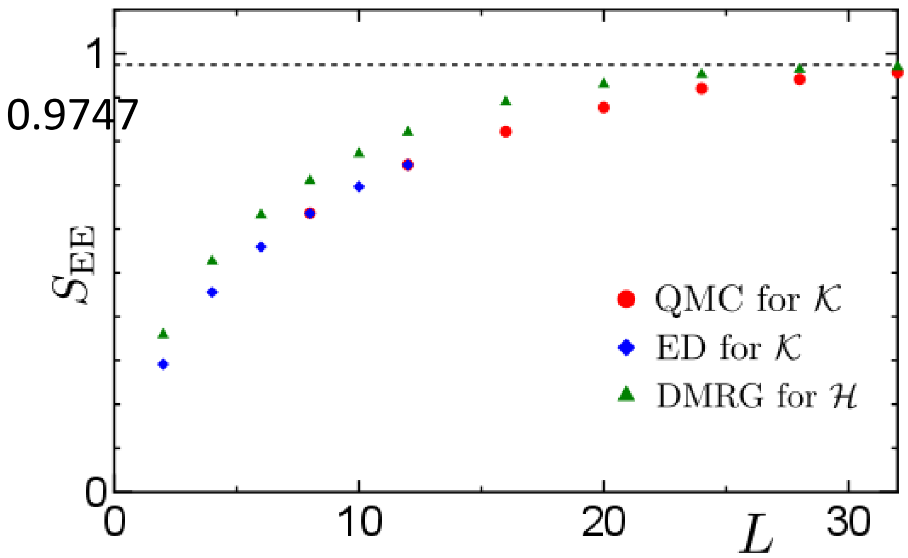
Cv

Fitting: Gaussian Kernel method

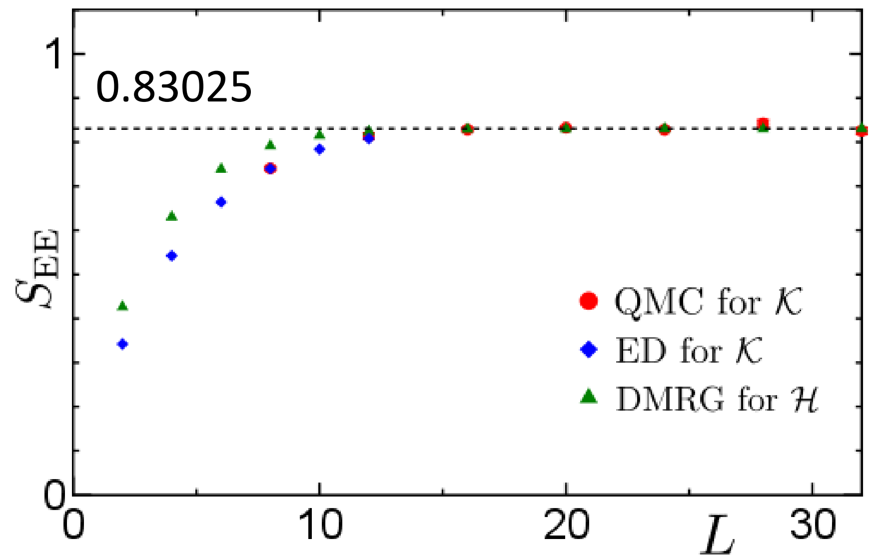


Eneanglement Entropy

$\Delta = 2.0$



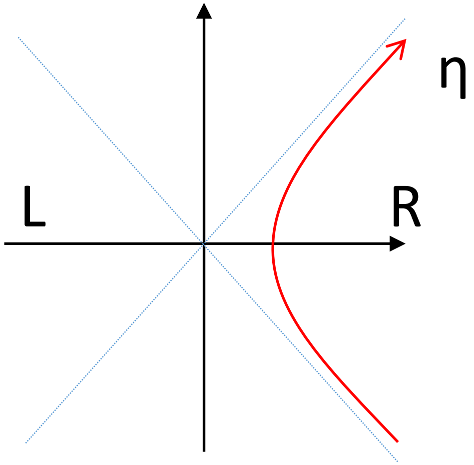
$\Delta = 3.0$



Estimation of EE approaches to the exact value of EE for the half0infinite subsystem

The deviation from the DMRG result originates from geometry of world sheets:
DMRG: cylinder,   corner Hamiltonian: annuls

Unruh-DeWitt detector



A harmonic oscillator coupled with a scalar field moving along the Rindler trajectory

$$S = \int d\eta \phi(x(\eta), t(\eta)) \hat{X}(\eta)$$

$$x = r\cosh(a\eta)\,,\, t = r\sinh(a\eta)$$

➡ The detector is excited by the thermalized vacuum.

Excitation rate is given by an integration of the Wightman function

➡ $$P_n \propto \int d\eta e^{i\omega_n \eta}\, {}_M\langle \phi(x(\eta), t(\eta)) \phi(r, 0) \rangle_M$$

Capturing the Bose distribution with the Unruh temp.

$$P_n \propto \frac{1}{e^{\beta_U \omega_n} - 1}$$

(massless case)

XXZ-chain analogue of the detector

A harmonic oscillator coupled with a spin in the XXZ chain

But, the detector does not accelerate in the chain literally .

Scalar field

$$\phi(x(\eta), t(\eta)) = e^{ia\eta L}\phi(r, 0)e^{-ia\eta L}$$

η-dependent Lorentz transformation

Spin coupled with the detector : $\mathcal{K}$ lattice Lorentz boost

$$\Rightarrow \quad S_n^{\mu}(\eta) = e^{-ia\eta\mathcal{K}} S_n^{\mu} e^{ia\eta\mathcal{K}}$$

$n \sim r$: distance from the entangle point

Autocorrelation function with respect to τ

$$G_n^{\mu}(\eta) \equiv \frac{\mathrm{Tr}\, S_n^{\mu}(\eta) S_n^{\mu}(0) e^{-\beta_\lambda \mathcal{K}}}{Z}$$

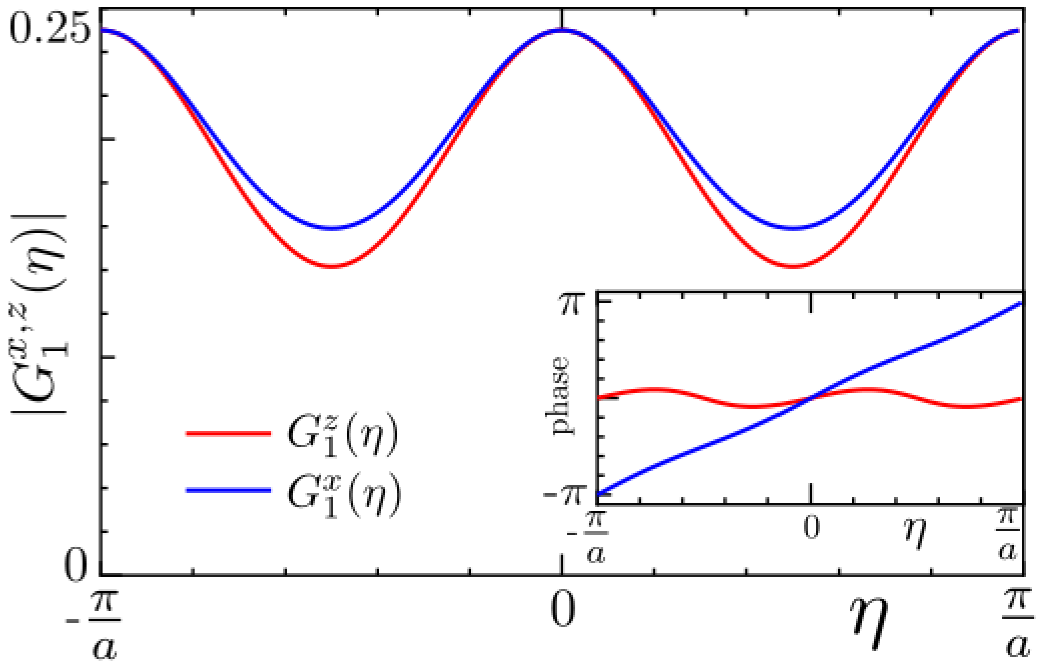
Autocorrelations

DMRG: Renormalization transformation matrix gives

the relation between the $\mathcal{K}$ diagonal bases and the usual spin bases

Bogoliubov trans. (Rindler) (Minkowski)

n=1



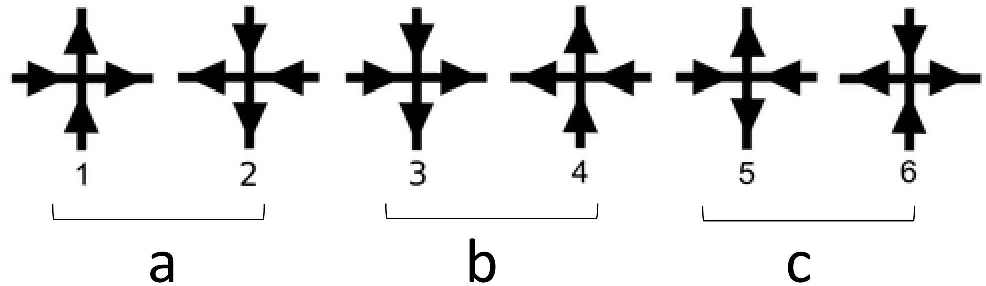
classical value

π/a periodicity

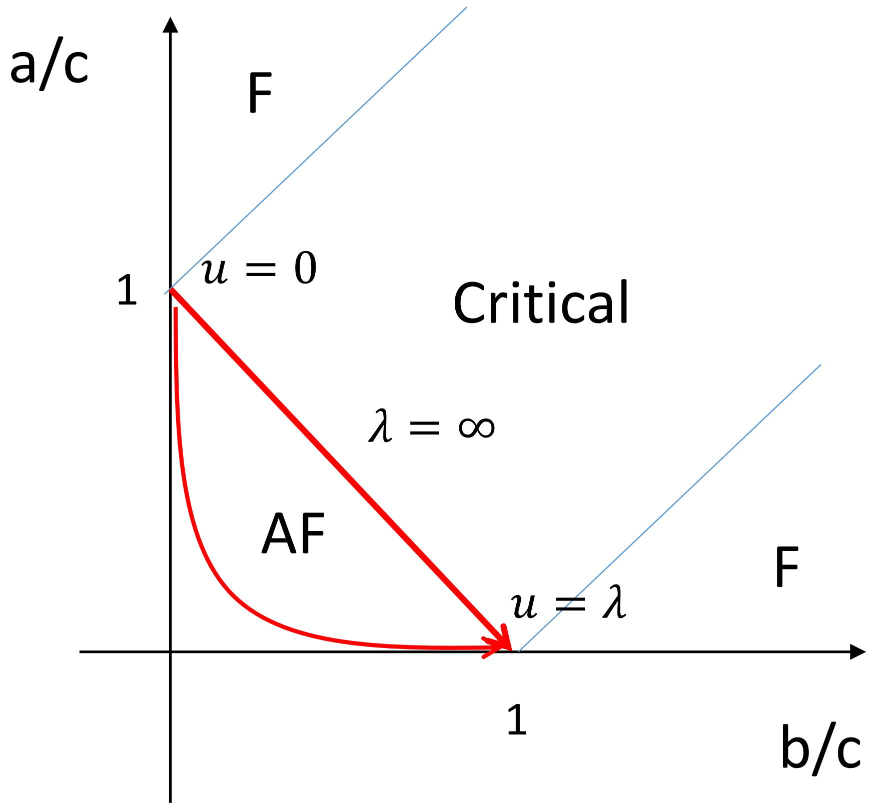
Imaginary shift
of the rapidity
+
lattice effect

rapidity space and 6-vertex model

(spectral parameter)



Phase diagram



$$0 < \Re u < \lambda \to 2\lambda$$

$$0 < \tau < \frac{2\pi}{a}$$

Imaginary part: Bethe ansatz

$$e^{ik} = \frac{\sinh((\lambda + i\alpha)/2)}{\sinh((\lambda - i\alpha)/2)}$$

$$\alpha = a\eta \quad \Rightarrow \quad -\frac{\pi}{a} < \eta < \frac{\pi}{a}$$

summary/discussions arXiv:1906.10441

- We calculate the groundstate properties of the Ising-like XXZ chain with a finite temperature formulation based on the entanglement Hamiltonian/CTM.

  Lattice Unruh effect

- We can understand the entanglement from the viewpoint of classical world lines surrounding the entangle point

  world-line entanglement

- Can we realize lattice Unruh-Dewitt detector?

  entanglement detector

- Critical cases? CFT, SSD, numerically bad convergence