Holographic duals of inhomogenous systems; Rainbow chain and SSD

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Introduction

- Inhomogeneous quantum many-body systems (on a lattice):

$$H = \sum_i h_{i,i+1} \implies H = \sum_i f(x_i, x_{i+1}) h_{i,i+1}$$

- Entanglement (Rindler) Hamiltonian: $f(x) = \frac{R^2 - x^2}{2R}$

- Sine-square deformation (SSD):
$f(x) = \cos\frac{2\pi x}{L} + 1 = \sin^2\frac{\pi(x - L/2)}{L}$ [Gendiar-Krcmar-Nishino (08), Hikihara-Nishino (11), ...]

- Möbius evolution: $f(x) = \cos\frac{2\pi x}{L} + \sqrt{1 - const./L^2}$
[Ishibashi-Tada (15-16); Okunishi (16); Wen-SR-Ludwig (16)]

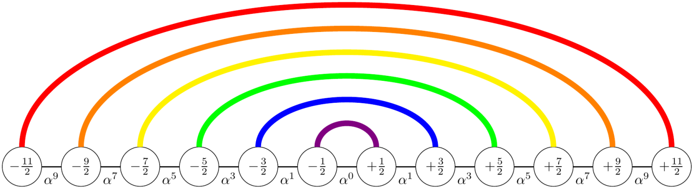
- Rainbow chain: $f(x) = e^{-h|x|}$

Rainbow chain

- [Vitagliano-Riera-Latorre (10), Ramirez-Rodriguez-Laguna-Sierra (14)]

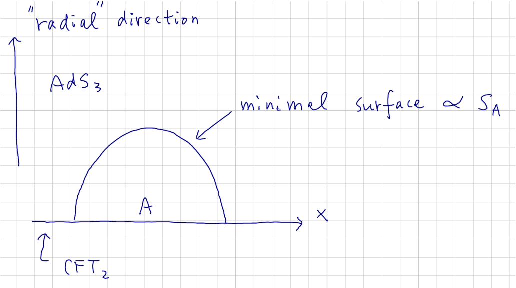
$$H = -c^\dagger_{1/2} c_{1/2} - \sum_{i=1/2}^{L-3/2} e^{-hi} \left[c^\dagger_i c_{i+1} + c^\dagger_{-i} c_{-i-1} \right]$$

- Concentric singlet formation
- Volume law entanglement for the half-chain partition: $S_A \sim L$.
- In the continuum, CFT on AdS_2.
  [Rodriguez-Laguna-Dubail-Ramirez-Calabrese-Sierra (16)]



- In this talk, I will develop holographic descriptions
- Of particular interest: the scaling of the entanglement entropy (at zero and finite T).

AdS/CFT, AdS_3/CFT_2 in particular



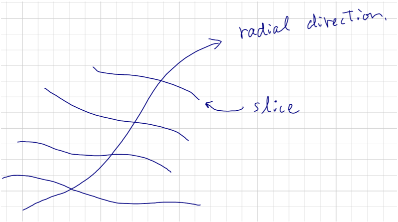
- Gravity in bulk AdS $\Leftrightarrow$ CFT on ∂AdS
- "Radius" R of AdS $\Leftrightarrow$ central charge c: $c = 3R/(2G_N)$
- BTZ black hole $\Leftrightarrow$ finite T

AdS/CFT, AdS_3/CFT_2 in particular

- *Kinematical*: Any stuff determined solely by conformal symmetry in CFT should have geometric descriptions in AdS. E.g., entanglement entropy for a single interval
- *Dynamical*: Einstein gravity in AdS realizes large c CFT. E.g., operator content, mutual information.

Different time-evolution $\leftrightarrow$ Different foliations

- Key concept and strategy: foliations (slicing)
- (Boundary metric $\rightarrow$ Einstein equation $\rightarrow$ Bulk geometry)
- Why different coord systems lead to different physics? (Diffeo invariance?) $\Leftarrow$ CFT is defined on a cutoff surface



Example: Entanglement (Rindler) Hamiltonian

- Rindler coord. $(u > 0, -\infty < t' < \infty)$:

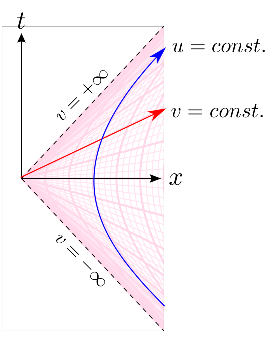
$$t = u \sinh(ht'), \quad x = u \cosh(ht')$$

- The half of the space(time) is inaccessible ("traced out"); the state is mixed at finite Unruh temperature $T = h/(2\pi)$.

- Metric:

$$ds^2_{Rindler} = -u^2 dt'^2 + du^2 = e^{2hx'}(-dt'^2 + dx'^2)$$

(Tortoise coordinate by $u =: h^{-1} e^{hx'}$.)



[Figures: Wikipedia]

Example: Entanglement (Rindler) Hamiltonian

- The Rindler Hamiltonian,

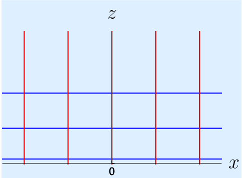
$$H_{Rindler} = \int_0^\infty du\, u\, \mathcal{H}(u).$$

- Bulk metric:

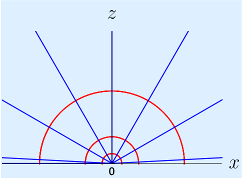
$$\begin{aligned} ds^2_{AdS_3} &= R^2 \frac{dz^2 + dx^2 - dt^2}{z^2} \\ &= \frac{R^2}{z^2}\left[(-u^2 dt'^2 + du^2) + dz^2\right] \\ &= \frac{R^2}{z^2}\left[e^{2hx'}(-dt'^2 + dx'^2) + dz^2\right]. \end{aligned}$$

- There are two asymptotic boundaries (hence two CFTs), which are entangled.
- (The entanglement Hamiltonian of the finite interval can be discussed similarly.

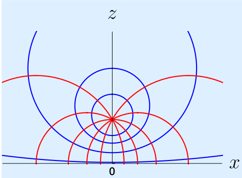
Different foliations ↔ Different time-evolution



Flat



Rainbow



Möbius (SSD)

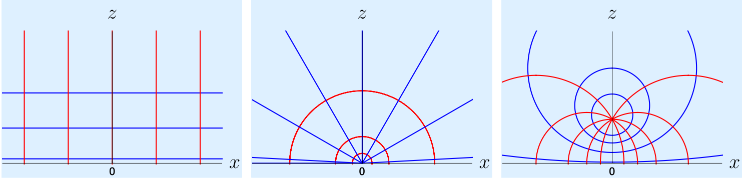
Bulk metric:

$$ds^2_{AdS_3} = R^2 \frac{dz^2 + dx^2 - dt^2}{z^2}$$

$$ds^2_{AdS_3} = \left[\frac{h^2 R^2}{\cos^2(h\Theta)}\right] \left[d\Theta^2 + \frac{1}{h^2\eta^2}\left(d\eta^2 - dt^2\right)\right]$$

$$ds^2_{AdS_3} = \frac{1}{\sinh^2 u}\left[du^2 + dv^2 - a^{-2}(\cosh u - \cos v)^2 dt^2\right]$$

Different foliations $\leftrightarrow$ Different boundary metrics



Flat Rainbow Möbius (SSD)

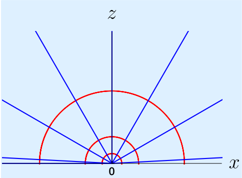
Slice metric:

$$ds^2_{Mink} = dx^2 - dt^2$$

$$ds^2_{AdS_2} = \frac{1}{h^2\eta^2}\left(d\eta^2 - dt^2\right)$$

$$ds^2_{Mobius} = -\left(1 - \tanh 2\gamma \cos\frac{2\pi x}{L}\right)^2 dt^2 + dx^2$$

Rainbow slicing



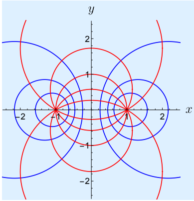
$$ds^2_{AdS_3} = \left[\frac{h^2 R^2}{\cos^2(h\Theta)}\right]\left[d\Theta^2 + \frac{1}{h^2\eta^2}\left(d\eta^2 - dt^2\right)\right]$$

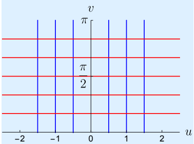
- There are two asymptotic boundaries (two CFTs) (similar to Rindler foliation)
- The two CFTs are connected at $(z, x) = 0$.
- Previously used, e.g., for AdS/BCFT [Takayanagi(11)]

C.f. Entanglement Hamiltonian and SSD in 2d CFT

- Conformal transformation:

$$w(z) = \log(z + R) - \log(z - R)$$





- EE hamiltonian on $[-R, +R] \to$ Hamiltonian with boundaries
- Transforming from strip to plane:

$$H = \int du\, T_{vv}|_{v_0=\pi} = \int_{-R}^{+R} dx\, \frac{(x^2 - R^2)}{2R} T_{yy}|_{y=0}$$

E.g., Casini-Huerta-Myers (11), Cardy-Tonni (16)

C.f. Entanglement Hamiltonian and SSD in 2d CFT

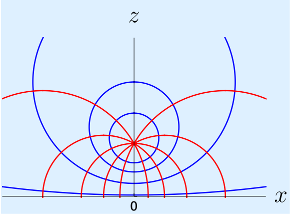
- Evolution in the "orthogonal direction" to the modular flow: = SSD.
- Evolution operator:

$$H = \int_0^{\pi} dv\, T_{uu}(u_0, v) = r_0^2 \int_0^{2\pi} d\theta\, \frac{\cos\theta + \cosh u_0}{\sinh u_0}\, T_{rr}(r, \theta)$$

- In the limit $R \to 0$,

$$H \sim \int_0^{L} ds\, \sin^2\left(\frac{\pi s}{L}\right) T_{rr}\left(\frac{L}{2\pi}, \frac{2\pi s}{L}\right)$$

Möbius foliation of AdS_3



- t-independent coord. transformation:

$$u + iv = \log(z + ix + a) - \log(z + ix - a) \stackrel{a \to 0}{\to} \frac{a}{z + ix}$$

- Metric:

$$ds^2_{AdS_3} = R^2 \frac{dz^2 + dx^2 - dt^2}{z^2} = \frac{R^2}{\sinh^2 u} \left[du^2 + dv^2 - a^{-2} (\cosh u - \cos v)^2 dt^2 \right]$$
$$\to \frac{R^2}{u^2} \left[dv^2 - a^{-2} (u^2 + v^2) dt^2 + du^2 \right]$$

The slice metric agrees with the one identified in [Wen-Wu (18)]

Finite temperature

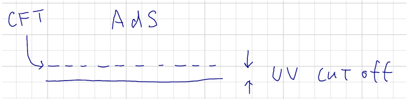
- Start from the finite T (holographic) EE:

$$S_A(x_1, x_2; \beta) = \frac{c}{3} \log \left[\frac{\beta}{\pi \sqrt{\epsilon_1}\sqrt{\epsilon_2}} \sinh\left(\frac{\pi(x_2 - x_1)}{\beta} \right) \right].$$

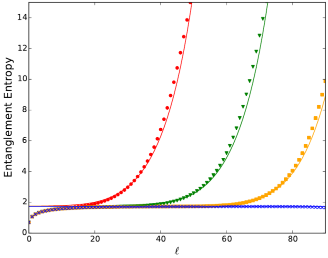
- Replace ϵ_1 and ϵ_2 with appropriate curvilinear cutoffs.

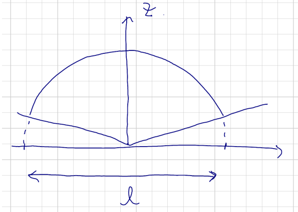
$$S_A(v_1, v_2; \beta) = \frac{c}{3} \log \left[\frac{\beta}{\pi\epsilon} \frac{\sinh\left(\frac{\pi}{\beta}(x(u=0,v_2) - x(u=0,v_1))\right)}{\sqrt{\frac{\partial z(u=0,v_1)}{\partial u}\frac{\partial z(u=0,v_2)}{\partial u}}} \right].$$

- Good approximation when (length of interval) $\ll T$.



EE for Rainbow chain

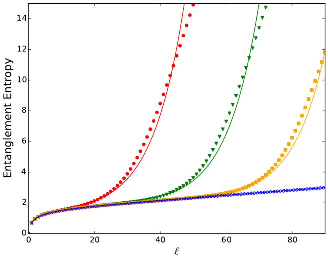


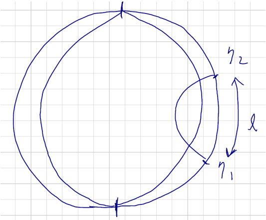


- "Defect" entanglement entropy

$$S_A(x;\beta,\epsilon) = \frac{c}{3}\log\left[\frac{\beta}{\pi h e^{h\ell}}\sinh\left(\frac{2\pi\epsilon e^{h\ell}}{\beta}\right)\right] + \cdots ,$$

EE for Rainbow chain

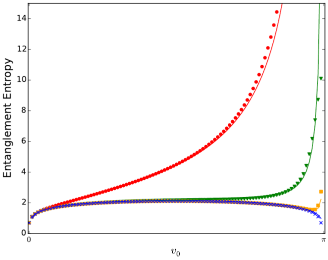


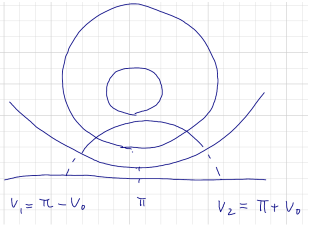


- "Half-chain" entanglement entropy

$$S_A(\ell;\beta,\eta_1) = \frac{c}{3}\log\left[\frac{\beta}{\epsilon\pi h\eta_1 e^{h\ell/2}}\sinh\left(\frac{\pi\eta_1(e^{h\ell}-1)}{\beta}\right)\right].$$

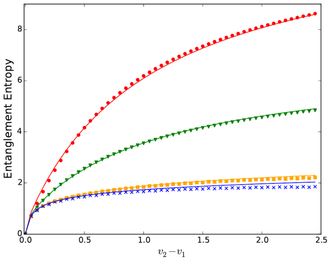
EE for Möbius evolution

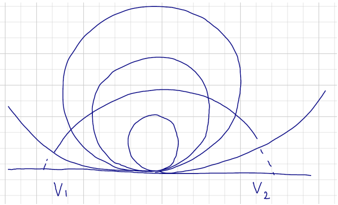




$$S_A(\pi - v_0, \pi + v_0; \beta) = \frac{c}{3} \log \left[\frac{4\beta}{L\epsilon} \cos^2\left(\frac{v_0}{2}\right) \sinh\left(\frac{L}{\beta} \tan\left(\frac{v_0}{2}\right)\right) \right].$$

EE for SSD evolution





$$S_A = \frac{c}{3} \log \left[\frac{\beta |v_1 v_2|}{2\pi a \epsilon} \sinh \left(\frac{2\pi a}{\beta} \left| \frac{v_2 - v_1}{v_1 v_2} \right| \right) \right].$$

Summary and outlook

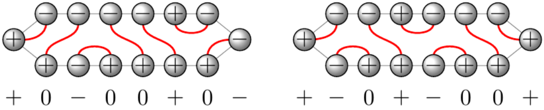
- Construction of holographic duals of rainbow chain and SSD.
- Computation of finite T entanglement entropy

Issues:

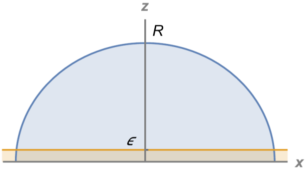
- Other inhomogenous systems?
- Other quantities, time-dependent setup ...
  Negativity and local quench [MacCormack-Kudler-Flam-SR]
- Higher dimensions ?

Rainbow chain, SPT, BCFT

- Folding rainbow chain → SPT phase [Nadir Samos Sáenz de Buruaga et al(18)]



- SPT phases ↔ BCFT [Qi-Katsura-Ludwig (11), Cho-Shiozaki-SR-Ludwig (16)]
- Rainbow foliation is closely related to BCFT.



[Picture: Cavalcanti et al.(18)]