タキオン真空とサイン二乗変形

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(理化学研究所, 和光)

PTEP, 2018, 12, 123B04 [arXiv:1809.01885 [hep-th]]
with Isao Kishimoto, Tomomi Kitade

はじめに

"Dipolar quantization and the infinite circumference limit of two-dimensional conformal field theories",
Nobuyuki Ishibashi, Tsukasa Tada
Int. J. Mod. Phys. A 31, 1650170 (2016) [arXiv:1602.01190v1]

The present formulation was also partially guided by previous approaches in the study of string field theory (SFT) [37, 38]*. It would be interesting if one can find more direct connections between the present result and SFT treatment, especially in the context of understanding the transition between open and closed strings* [39]*.*

[37] M. Kiermaier, A. Sen and B. Zwiebach, JHEP0803, 050 (2008) [arXiv:0712.0627 [hep-th]].
[38] T. Takahashi and S. Zeze, Prog. Theor. Phys.110, 159 (2003) [hep-th/0304261].
[39] T. Takahashi, Prog. Theor. Phys. Suppl.188, 163 (2011).

理研研究会「サイン 2 乗変形（SSD）とその周辺」
日時: 2017 年 6 月 30 日 10:30 - 17:20
会場: セミナー室 (160 号室)

印象に残ったこと

SSD mechanism

· $\left\langle v_{\mathrm{SSD}} \,\middle|\, v_{\mathrm{PBC}} \right\rangle = 1$

⇒ 帰ってから Mathematica で確かめた! (驚きました)

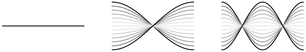
· $\mathcal{H}_{\mathrm{SSD}} = \frac{1}{2}\mathcal{H}_0 - \frac{1}{4}(\mathcal{H}_+ + \mathcal{H}_-), \qquad \mathcal{H}_\pm |0\rangle = 0$

· $H = \frac{1}{2}L_0 - \frac{1}{4}(L_1 + L_{-1}), \qquad L_0 |0\rangle = L_{\pm 1} |0\rangle = 0$

⇒ 異なった言い方ができるはず?

ボゾン型開弦のタキオン

開弦の固有状態



	状態	(質量)^2	スピン	成分数
タキオン	$\|p\rangle$	$-\dfrac{1}{\alpha'}$	0	1
ベクトル粒子	$\alpha^i_{-1}\,\|p\rangle$	0	1	$D-2=24$
テンソル粒子	$\alpha^i_{-1}\alpha^j_{-1}\,\|p\rangle$ $(p^i\alpha^j_{-2}-p^j\alpha^i_{-1})\,\|p\rangle$	$+\dfrac{1}{\alpha'}$	2	$\frac{D(D-2)}{2}=276$

(臨界次元: $D=26$)

$$p^{j_1}\cdots p^{j_M}\alpha^{i_1}_{-n_1}\cdots\alpha^{i_N}_{-n_N}\,|p\rangle\,,\qquad M^2=\Big(\sum_{k=1}^{N}n_k-1\Big)/\alpha'$$

$$\left[\,\alpha^i_n,\,\alpha^j_m\,\right]=n\delta_{n+m,0}\delta^{i,j}$$

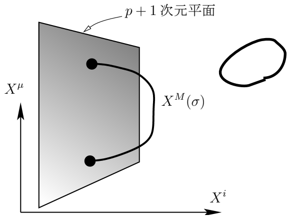
D ブレーン

Polchinski '95

D ブレーンとは、開弦の端点がくっつく $p+1$ 次元の超曲面

($p+1 \leq 26$)

- $X^\mu(\sigma)$ $(\mu = 0, \cdots, p)$: ノイマン境界条件
  $X^i(\sigma)$ $(i = p+1, \cdots, 25)$: ディリクレ境界条件
- D ブレーン自身が力学的な対象
- 開弦は D ブレーンのゆらぎを表す
- タキオンの存在はボゾン型 D ブレーンの不安定性を示す
- バルク時空には閉弦が存在する



弦の場の理論 (String Field Theory)

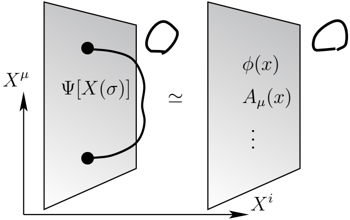
場 $\phi(x)$ を拡張した弦の場 $\Psi[X(\sigma)]$ を力学変数とする理論

Witten '86

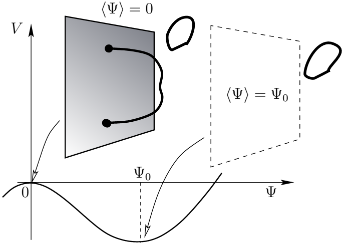
$$S[\Psi] = \frac{1}{g^2}\int\left(\frac{1}{2}\Psi * Q_B\Psi + \frac{1}{3}\Psi*\Psi*\Psi\right)$$

$$= \frac{1}{g^2}\int d^{p+1}x\left\{-\frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2\alpha'}\phi^2 - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{3}\kappa^3\left(e^{\log\kappa\,\alpha'\partial^2}\phi\right)^3 + \cdots\right\}$$

$$(\kappa = 3\sqrt{3}/4)$$



タキオン真空



Sen '99, Sen-Zwieback '00,· · ·

T.T-Tanimoto '02, Kishimoto-T.T '02, · · ·

Schnabl '05, Erler-Scnabl '09, T.T-Zeze '03, · · ·

Ishibashi '14, Kishimoto-Masuda-T.T '14, · · ·

- 弦の場の理論には安定なタキオン真空が存在
- タキオン真空では、D ブレーンが消滅
  (Ψ_0 に対するエネルギーが、D ブレーンのエネルギーを相殺)
- タキオン真空では、開弦の場のゆらぎはゲージ自由度となる
  (D ブレーンのゆらぎの自由度が消えている)

タキオン真空上での" 開弦" のハミルトニアン

Takahashi ('03), Takahashi-Zeze ('03)

$$
\begin{aligned}
L' &= \frac{1}{2}L'_0 - \frac{1}{4}(L'_2 + L'_{-2}) + \frac{3}{2} \\
&= \oint \frac{dz}{2\pi i}\frac{-1}{4z}(z^2-1)^2\, T'(z) + \cdots \\
&= \int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} \sin^2\sigma\, T'(\sigma) + \cdots
\end{aligned}
$$

タキオン真空上の開弦のハミルトニアンにサイン二乗変形が表れている

$\Leftarrow$ 石橋-多田の指摘!

SSD

開いた一次元系をサイン二乗変形した系の基底状態は、周期的境界条件を課した系の基底状態と一致している。

SFT

タキオン真空では、D ブレーンが消滅しているので開弦のタキオン状態は基底状態ではなくなり、バルクに閉弦が存在するので閉弦の基底状態がこの系の基底状態になるはずだ!

予 想

サイン二乗変形 $\simeq$ タキオン真空

1 次元離散フェルミオン系

"*Field Theories of Condensed Matter Physics*", E. Fradkin

1 次元フェルミオン系

$$\hat{H}_{\mathrm{O}} \quad = \quad -t\sum_{l=0}^{N}\left(\hat{c}_l^{\dagger}\hat{c}_{l+1}+\hat{c}_{l+1}^{\dagger}\hat{c}_l\right).$$

$\hat{c}_l$ を

$$\hat{c}_l \equiv e^{i\frac{\pi}{2}l}a_l \tag{3.1}$$

と変換すると、ハミルトニアンは

$$\hat{H}_{\mathrm{O}} = -it\sum_{l=1}^{N}a_l^{\dagger}(a_{l+1}-a_{l-1}), \qquad (a_0 = a_{N+1} = 0)$$

$$\{a_l,\, a_{l'}^{\dagger}\} = \delta_{l,l'}, \quad \{a_l,\, a_{l'}\} = \{a_l^{\dagger},\, a_{l'}^{\dagger}\} = 0$$

さらに、

$$\varphi_l^1 \equiv \frac{1}{\sqrt{2}}(a_l + a_l^\dagger), \quad \varphi_l^2 \equiv \frac{1}{\sqrt{2}i}(a_l - a_l^\dagger),$$

として、エルミート演算子 $\varphi_l^\mu \quad (\mu = 1, 2)$ を導入すると、

$$\hat{H}_{\mathrm{O}} = -it \sum_{l=0}^{N} \left(\varphi_l^1 \varphi_{l+1}^1 + \varphi_l^2 \varphi_{l+1}^2 \right) \quad (\varphi_0^\mu = \varphi_{N+1}^\mu = 0)$$

$$\{\varphi_l^\mu, \varphi_{l'}^\nu\} = \delta_{l,l'} \delta^{\mu\nu}$$

のように、独立な演算子 φ_l^1 と φ_l^2 に対して対称な形に書きなおすことができる。

この系の連続極限について考えていこう。一つの自由度を抜き出し、それを φ_l として考える。
まず、その 1 自由度に対するハミルトニアンを、偶数サイトの演算子と奇数サイトの演算子に分けて

$$\hat{H}_{\mathrm{O}} = -\frac{it}{2} \sum_{k=1}^{\left[\frac{N+1}{2}\right]} \{\varphi_{2k-1}(\varphi_{2k} - \varphi_{2k-2}) + \varphi_{2k}(\varphi_{2k+1} - \varphi_{2k-1})\}$$

と書く。ここで、$\xi_k = \varphi_{2k-1}, \quad \eta_k = \varphi_{2k}$ という演算子を導入すると、反交換関係は

$$\{\xi_k, \xi_{k'}\} = \delta_{k,k'}, \quad \{\eta_k, \eta_{k'}\} = \delta_{k,k'}, \quad \{\xi_k, \eta_{k'}\} = 0$$

であり、ハミルトニアンは

$$\hat{H}_{\mathrm{O}} = -\frac{it}{2} \sum_{k=1}^{\left[\frac{N+1}{2}\right]} \{\xi_k(\eta_k - \eta_{k-1}) + \eta_k(\xi_{k+1} - \xi_k)\}$$

となる。

連続極限をとると、$N \to \infty$ の極限をとると、連続極限をとったハミルトニアンが

$$\hat{H}_{\mathrm{O}} = -\frac{i}{2\pi}\int_0^{\pi}\left(\xi\eta' + \eta\xi'\right)d\sigma$$

となり、反交換関係が

$$\begin{aligned}&\{\xi(\sigma),\,\xi(\sigma')\} = \pi\delta(\sigma-\sigma'),\quad \{\eta(\sigma),\,\eta(\sigma')\} = \pi\delta(\sigma-\sigma'),\\ &\{\xi(\sigma),\,\eta(\sigma')\} = 0,\end{aligned}$$

となる。
境界条件は、N を偶数に保ちながら極限をとる場合と、奇数に保ちながら極限をとる場合とで異なってくる。
N が偶数の場合、格子上の変数の境界条件は $\varphi_0 = \varphi_{N+1} = 0$ であるが、$N+1$ が奇数となるため、連続変数の境界条件が

$$\eta(0) = 0, \qquad \xi(\pi) = 0$$

となる。同様に考えて、N が奇数の場合の境界条件は

$$\eta(0) = 0, \qquad \eta(\pi) = 0$$

となる。

弦理論との対応をより明確に見るために、

$$\psi_{\pm} \equiv \frac{1}{\sqrt{2}}(\xi \pm \eta)$$

と定義すると、ハミルトニアンが

$$\hat{H}_{\mathrm{O}} = \frac{i}{2\pi}\int_0^{\pi} \left(\psi_+\partial_\sigma\psi_+ - \psi_-\partial_\sigma\psi_-\right) d\sigma$$

となり、反交換関係が

$$\{\psi_+(\sigma),\, \psi_+(\sigma')\} = \pi\delta(\sigma-\sigma'), \quad \{\psi_-(\sigma),\, \psi_-(\sigma')\} = \pi\delta(\sigma-\sigma'),$$
$$\{\psi_+(\sigma),\, \psi_-(\sigma')\} = 0,$$

となる。
これは、２次元マヨラナフェルミオン系に対するハミルトニアンであり、開いた超弦理論のフェルミオン部分に対応していることがわかる。
境界条件は、N が偶数の場合、

$$\psi_+(0) = \psi_-(0), \quad \psi_+(\pi) = -\psi_-(\pi)$$

となり、Neveu-Schwarz 型の境界条件である。N が奇数の場合は、

$$\psi_+(0) = \psi_-(0), \quad \psi_+(\pi) = \psi_-(\pi)$$

となり、Ramond 型の境界条件であることがわかる。

サイン二乗変形した系の連続極限

$$\hat{H}_{\mathrm{O}} = \frac{i}{2\pi}\int_0^\pi \sin^2\sigma\ (\psi_+\partial_\sigma\psi_+ - \psi_-\partial_\sigma\psi_-)\, d\sigma$$

N が偶数の場合、Neveu-Schwarz 型の境界条件

$$\psi_+(0) = \psi_-(0), \quad \psi_+(\pi) = -\psi_-(\pi)$$

N が奇数の場合、Ramond 型の境界条件

$$\psi_+(0) = \psi_-(0), \quad \psi_+(\pi) = \psi_-(\pi)$$

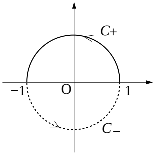
境界条件があるために、$\psi_+(\sigma)$ と $\psi_-(\sigma)$ が独立な演算子とならない！
$\Rightarrow$ 開弦の特徴！

なぜ、周期的境界条件をもつ系（閉弦系）を記述するのか?

Open string Hamiltonian

The Hamiltonian of an open string is given by

$$H_{\mathrm{O}} = \int_{C_+} \frac{dz}{2\pi i}\, zT(z) + \int_{C_-} \frac{dz}{2\pi i}\, zT(z),$$



$T(z)$: the energy-momentum tensor.

Each term corresponds to Hamiltonians of left and right moving modes, respectively, but they do not commute with each other due to open boundary conditions on $T(z)$.

The Hamiltonian is given by the zeroth component of the Virasoro operators: L_0. So, we do not encounter antiholomorphic Virasoro operators in the open string system.

Sine-squre-like deformation

Here, we consider the deformed Hamiltonian:

$$H_g = H_g^+ + H_g^-, \quad H_g^\pm = \int_{C_\pm} \frac{dz}{2\pi i}\, g(z) T(z),$$

where $g(z)$ is a holomorphic function satisfying $g(\pm 1) = \partial g(\pm 1) = 0$.

H_g^+ and H_g^- are left and right moving modes of H_g.

The simplest example of $g(z)$ is given by

$$g(z) = -\frac{1}{4z}(z^2-1)^2.$$

If we change the variable as $z = \exp(i\theta)$, the weighting function in H_g is changed to $z^{-1}g(z) = \sin^2\theta$. Hence, the deformed Hamiltonian provides a sort of generalization of the SSD Hamiltonian. In this sense, we call it the sine-square-like deformation, or SSLD for short.

$T(z)$ is expanded by holomorphic Virasoro operators only:

$$T(z) = \sum_{n=-\infty}^{\infty} L_n z^{-n-2}.$$

By using this expansion form and the Virasoro algebra, we can obtain a commutation relation of $T(z)$:

$$[T(z), T(z')] = -(\,T(z) + T(z')\,)\,\partial\delta(z,z') - \frac{c}{12}\partial^3\delta(z,z'),$$

where c is the central charge of $T(z)$.

By this equation, we can calculate the commutation relation between H_g^+ and H_g^-.

The important point is that surface terms appear in the calculation as a result of derivatives of the delta function and these terms include a singular factor $\delta(\pm 1, \pm 1)$.

However, the singular surface terms turn out to vanish due to the factors $g(\pm 1)$ and $\partial g(\pm 1)$, which are set to zero in the definition of H_g.

As a result, we find

$$\left[\, H_g^+,\, H_g^- \,\right] = 0$$

and then the deformed system is decomposed into the left and right moving parts as in periodic systems.

Accordingly, it is concluded that the deformed system described by H_g corresponds not to an open string system, but to a closed string system, although the Hamiltonian is constructed by a single holomorphic energy-momentum tensor.

It should be noted that **the zeros of $g(z)$ and $\partial g(z)$ at open string boundaries cause the decoupling of the left and right moving sectors!**

Now, we will illustrate equal-time contours generated by the Hamiltonian for the simplest function

$$g(z) = -\frac{1}{4z}(z^2 - 1)^2.$$

with a focus on emergence of left and right moving sectors.

According to Ishibashi-Tada, we introduce the parameters, t and s, into the worldsheet generated by H_g:

$$t + is = \int^z \frac{dz}{g(z)} = \frac{2}{z^2 - 1},$$

where t denotes time and s parameterizes a string at a certain time.

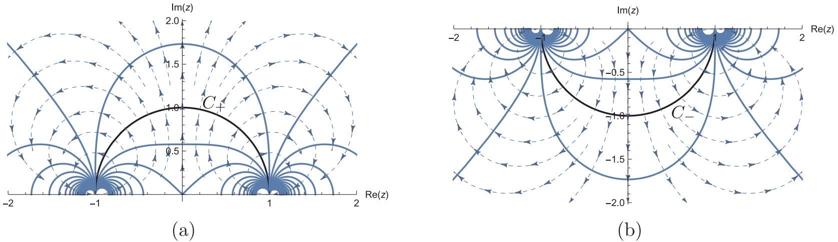
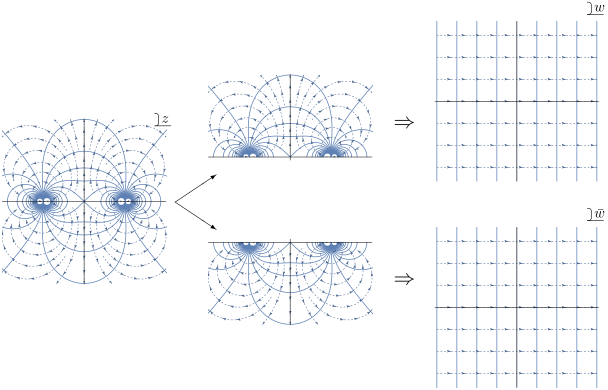


Figure: Equal-time contours on the z plane (solid lines). Dashed lines with arrows denote evolution of time t.

These contours have a remarkable feature that the string boundaries are fixed at $z = \pm 1$ during propagation of the string. One complex number $t + is$ corresponds to two points in the z plane.

Accordingly, we introduce a complex coordinate $w = t + is$ for the upper half z plane and $\bar{w} = t + is$ for the lower half plane.

By this mapping, the upper half plane corresponds to the whole w plane, and the lower half plane to the other $\bar{w}$ plane:



Hence, the equal-time contours by H_g lead us to the worldsheet which consists of two complex planes.

The two planes, w and $\bar{w}$, corresponding to the upper and lower half z planes are generated by the left and right moving Hamiltonian, H_g^+ and H_g^-, respectively.

Therefore, they can be regarded as holomorphic and antiholomorphic worldsheets of a closed string.

Now that we have obtained two decoupled Hamiltonians for the left and right moving sectors, we can construct two independent Virasoro operators according to Ishibashi-Tada:

$$\mathcal{L}_\kappa = \int_{C_+^t} \frac{dz}{2\pi i} g(z) f_\kappa(z) T(z), \qquad \tilde{\mathcal{L}}_\kappa = \int_{C_-^t} \frac{dz}{2\pi i} g(z) f_\kappa(z) T(z),$$

where $g(z)$ is the same function as that in the Hamiltonian H_g.
$f_\kappa(z)$ is defined by the differential equation

$$g(z) \frac{\partial}{\partial z} f_\kappa(z) = \kappa f_\kappa(z).$$

For a constant time t, C_+^t and C_-^t denote integral contours along the equal-time line on the upper and lower half z plane, respectively.

We should note again that $T(z)$ **including in $\mathcal{L}_\kappa$ and $\tilde{\mathcal{L}}_\kappa$ is the same energy-momentum tensor of the open string system.**

$\mathcal{L}_0$ and $\tilde{\mathcal{L}}_0$ provide the left and right moving parts of the Hamiltonian, that is, $\mathcal{L}_0 = H_g^+$ and $\tilde{\mathcal{L}}_0 = H_g^-$.

$\mathcal{L}_\kappa$ satisfies continuous Virasoro algebra:

$$[\mathcal{L}_\kappa, \mathcal{L}_{\kappa'}] = (\kappa - \kappa')\mathcal{L}_{\kappa+\kappa'} \\ + \frac{c}{12}\int_{C_+^t} \frac{dz}{2\pi i} \left\{ (\kappa - \kappa') \left(\frac{\partial^2 g}{\partial z^2} - \frac{1}{2g}\left(\frac{\partial g}{\partial z}\right)^2 \right) + \frac{\kappa^3 - {\kappa'}^3}{2g} \right\} f_{\kappa+\kappa'}(z).$$

Ishibashi-Tada '16

The right moving sector of the Virasoro operator $\tilde{\mathcal{L}}_\kappa$ can be also defined by integration along the integration path on the lower half plane. Similarly, $\tilde{\mathcal{L}}_\kappa$ satisfies the continuous Virasoro algebra.

Moreover, since C_+^t and C_-^t have no intersections, $\mathcal{L}_\kappa$ and $\tilde{\mathcal{L}}_\kappa$ commute with each other:

$$[\mathcal{L}_\kappa,\, \tilde{\mathcal{L}}_{\kappa'}] = 0.$$

Thus, we have found the **two independent Virasoro algebras in a deformed open string system, which can be regarded as the Virasoro algebras for closed strings, that is, the holomorphic and antiholomorphic parts.**

Kishimoto, Kitade and T.T ('18)

Energy-momentum tensor

We define an operator at the tachyon vacuum:

$$\begin{aligned}\mathcal{T}(z) &\equiv e^{-h(z)}\{Q',\, b(z)\} \\ &= T(z) + \partial h(z)\, j_{\mathrm{gh}}(z) - (\partial h(z))^2 + \frac{3}{2} e^{-h(z)} \partial^2 e^{h(z)}.\end{aligned}$$

We find that $\mathcal{T}(z)$ satisfies the same OPE as $T(z)$ with zero central charge:

$$\mathcal{T}(y)\mathcal{T}(z) \sim \frac{2}{(y-z)^2}\mathcal{T}(z) + \frac{1}{y-z}\partial\mathcal{T}(z).$$

Here, it should be noted that $\mathcal{T}(z)$ includes not only operators but also a function in its form.

Since $h(z)$ is related to a coordinate frame of worldsheets, $\mathcal{T}(z)$ has an explicit dependence on the frame.

Virasoro algebra

By using $\mathcal{T}(z)$, we can define the continuous Virasoro operator at the tachyon vacuum:

$$\mathcal{L}_\kappa \equiv \int_{C_+} \frac{dz}{2\pi i} g(z) f_\kappa(z) \mathcal{T}(z), \qquad \tilde{\mathcal{L}}_\kappa \equiv \int_{C_-} \frac{dz}{2\pi i} g(z) f_\kappa(z) \mathcal{T}(z),$$

where the weighting function is related to $h(z)$ as $g(z) = z e^{h(z)}$.

Since $e^{h(z)}$ has second order zeros at $z = \pm 1$, $g(z)$ also has second order zeros at $z = \pm 1$.

These operators satisfy the holomorphic and antiholomorophic continuous Virasoro algebra for $c = 0$. ($\mathcal{L}_0 = H_+$ and $\tilde{\mathcal{L}}_0 = H_-$.)

By definition of $\mathcal{T}(z)$, these operators commute with $Q'_\pm$:

$$[Q'_\pm, \mathcal{L}_\kappa] = [Q'_\pm, \tilde{\mathcal{L}}_\kappa] = 0.$$

Thus, we have found the continuous Virasoro algebra at the tachyon vacuum.

まとめ

開弦系のサイン二乗変形が閉じた系を表すのは、ハミルトニアンのゼロ点のために、ハミルトニアンの右向き部分とハミルトニアンの左向き部分が可換になるためである

ハミルトニアンの左右分離の結果、左右独立の連続ビラソロ代数が現れる

開弦の場の理論におけるタキオン真空上では、開弦のハミルトニアンにサイン二乗変形が現れる。

その結果、タキオン真空上での開弦の場の理論に、閉弦理論がもつ対称性を見出すことができる。

対称性の役割の大きさを考えれば、開弦の場の理論によって、閉弦の力学を解析できる可能性がある！

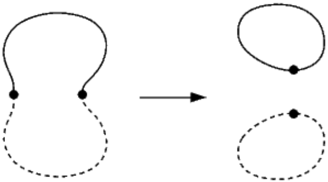


Figure: String pictures before and after SSLD. The solid and dashed lines correspond to holomorphic and antiholomorphic parts of a string. As a result of SSLD, open string boundaries (black dots) become joined and an open string divides to holomorphic and antiholomorphic strings.